LINEAR PERSPECTIVE:

OR, A

New METHOD

Of Reprefenting juftly all manner of

OBJECTS

As they appear to the E Y E

IN ALL

SITUATIONS.

A Work neceffary for PAINTERS,
ARCHITECTS, &c. to ~~Judge of, and~~
Regulate Defigns by.

BY

Brook Taylor, LL.D. and R. S. S.

TO THE

READER.

I N this Treatise I have endeavour'd to render the Art of PERSPECTIVE more general, and more easy, than has yet been done. In order to this, I find it necessary to lay aside the common Terms of Art, which have hitherto been used, such as Horizontal Line, Points of Distance, &c. and to use new ones of my own; such as seem to be more significant of the Things they express, and more a-
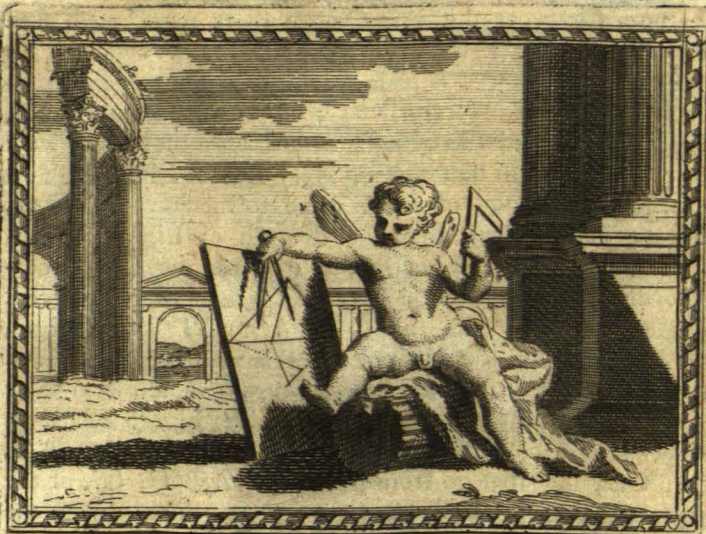greeable

greeable to the general *Notion* I have formed to my self of this *Subject*.

Thus much I thought *necessary* to say by way of Preface ; *because it always needs an Apology to change Terms of Art*, or *any way to go out of the common Road. But* I shall *add no more*, *because the shortness of the Treatise* it self *makes it needless to trouble the Reader with a more particular Account of it.*
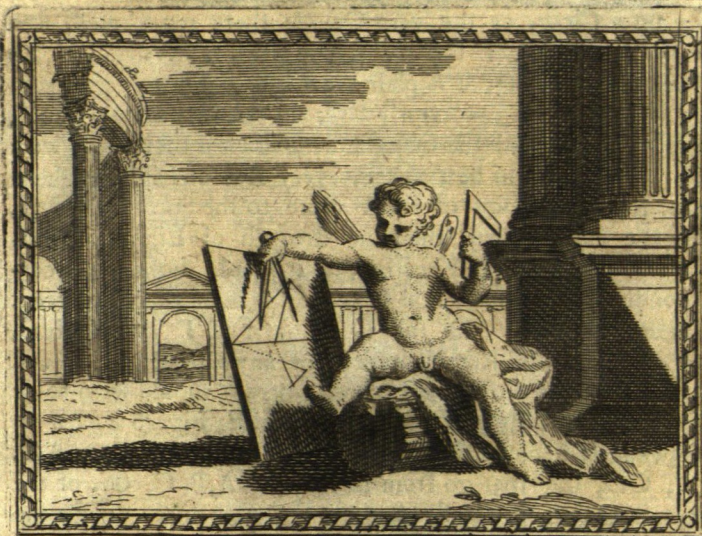
LINEAR PERSPECTIVE:

SECTION I.

Containing an Explanation of those things that are
necessary to be understood in order to the Practice
of PERSPECTIVE.

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LINEAR PERSPECTIVE:

SECTION I.

Containing an Explanation of those things that are necessary to be understood in order to the Practice of PERSPECTIVE.

PERSPECTIVE is the Art of drawing on a Plane the Appearances of any Figures, by the Rules of Geometry.

In order to understand the Principles of this Art, we must consider, That a Picture painted in its utmost degree of Perfection, ought so to affect the Eye of the Beholder, that he should not be able to judge, whether what he sees be only a few Colours laid artifici-

B   ally

ally on a Cloth, or the very Objects there reprefented, feen thro' the Frame of the Picture, as thro' a Window. To produce this Effect, it is plain the Light ought to come from the Picture to the Spectator's Eye, in the very fame manner, as it would do from the Objects themfelves, if they really were where they feem to be ; that is, every Ray of Light ought to come from any Point of the Picture to the Spectator's Eye, with the fame Colour, the fame ftrength of Light and Sha- dow, and in the fame Direction, as it would do from the cor- refponding Point of the real Object, if it were placed where it is imagined to be. So that (*Fig.* 1.) if E T be a Picture, and *a b c d* be the Reprefentation of any Object on it, and A B C D be the real Object placed where it fhould feem to be to a Spectator's Eye in O; then ought the Figure *a b c d* to feem exactly to cover the Figure A B C D, and the Rays A O, B O, C O, &c. that go from any Points A, B, C, &c. of the original Object to the Spectator's Eye O, ought to cut the Picture in the correfponding Points *a, b, c,* &c. of the Repre- fentation. Wherefore, in order to demonftrate any Propofiti- on in this Treatife, upon this Principle, I always fuppofe the real original Object to be placed where it fhould appear to be.

DEFINITIONS.

DEF. I.

THE Center of the Picture is that Point where a Line from the Spectator's Eye cuts it (or its Plane continued beyond the Frame, if need be) at Right Angles.

If the Plane C D (*Fig.* 2.) be the Picture, and O the Spectator's Eye, then a Perpendicular let fall on the Picture from O, will cut it in its Center P.

DEF.

D E F. II.

The Diſtance of the Picture, or principal Diſtance, is the Diſtance between the Center of the Picture and the Spectator's Eye.

In the ſame Figure P O is the Diſtance of the Picture.

D E F. III.

The Interſection of an Original Line is that Point where it cuts the Picture.

If I K be an Original Line cutting the Picture in C, then is C the Interſection of the Line I K.

D E F. IV.

The Interſection of an Original Plane, is that Line wherein it cuts the Picture.

A P is an Original Plane cutting the Picture in the Line C Q, which therefore is its Interſection.

D E F. V.

The Vaniſhing Point of an Original Line, is that Point where a Line paſſing thro' the Spectator's Eye, parallel to the Original Line, cuts the Picture.

Such is the Point V, the Line Q V being parallel to the Original Line I K.

B 2 *COROL*

COROL. I.

Hence it is plain, that Original Lines, which are parallel to each other, have the same Vanishing Point. For one Line passing thro' the Spectator's Eye, parallel to them all, produces the Vanishing Point of 'em all, by this Definition.

COROL. 2.

Those Lines that are parallel to the Picture have no Vanishing Points. Because the Lines which should produce the Vanishing Points, are in this Case also parallel to the Picture, and therefore can never cut it.

COROL. 3.

The Lines that generate the Vanishing Points of two Original Lines, make the same Angle at the Spectator's Eye, as the Original Lines do with each other.

DEF. VI.

The Vanishing Line of an Original Plane, is that Line wherein the Picture is cut by a Plane passing thro' the Spectator's Eye parallel to the Original Plane.

Such is the Line V S, the Plane E F, being parallel to the Original Plane A B.

COROL. I.

Hence Original Planes, that are parallel, have the same Vanishing Line. For one Plane passing thro' the Spectator's Eye, parallel to them all, produces that Vanishing Line.

COROL.

COROL. 2.

All the Vanishing Points of Lines in parallel Planes, are in the Vanishing Line of those Planes. For the Lines that produce those Vanishing Points, (by *Def.* 5.) are all in the Plane that produces that Vanishing Line, (by this *Def.*)

COROL. 3.

The Planes which produce the Vanishing Lines of two Original Planes, being parallel to the Original Planes, and passing both thro' the Spectator's Eye, (by this *Def.*) have their common Intersection passing thro' the Spectator's Eye, parallel to the Intersection of the Original Planes, and are inclined to each other in the same Angle as the Original Planes are. And hence,

COROL. 4.

The Vanishing Point of the common Intersection of two Planes, is the Intersection of the Vanishing Lines of those Planes,

COROL. 5.

The Vanishing Line, and Intersection of the same Original Plane, are parallel to each other. Because they are generated by parallel Planes. (By this *Def.* and *Def.* 4.)

DEF. VII.

The Center of a Vanishing Line is that Point where it is cut by a Perpendicular from the Spectator's Eye.

Such is S, O S being perpendicular to the Vanishing Line V S.

C O R O L.

A Line drawn from the Center of the Picture, to the Center of a Vanishing Line, is perpendicular to that Vanishing Line, As in the Line P S, to the Vanishing Line S V.

D E F. VIII.

The Distance of a Vanishing Line is the Distance between its Center, and the Eye of the Spectator, As O S.

C O R O L.

The Distance of a Vanishing Line is the Hypothenuse of a Right-Angled Triangle, (such as O P S) whose Base being the Principal Distance (O P) its Perpendicular is the Distance (P S) between the Center of the Picture, and the Center of that Vanishing Line.

D E F. IX.

The Directing Plane is a Plane passing thro' the Spectator's Eye parallel to the Picture.

Such is the Plain G H.

D E F. X.

The Directing Point of an Original Line is that Point where it cuts the Directing Plane.

Such is the Point G, to the Original Line I K.

P R O P.

PROP. 1. THEOR. 1.

The Representation of a Line is Part of a Line paſ-
ſing thro' the Interſection and Vaniſhing Point of
the Original Line.

(*Fig.* 2.) For the viſual Rays I O, K O, &c. which pro-
duce the Repreſentations of the Points I, K, &c. of the Ori-
ginal Line I K, by their Interſections with the Picture in *i*, *k*,
&c. are all in a Plane paſſing thro' the Original Line I K, and
the Spectator's Eye O. But the Point C, which is the Inter-
ſection of the Original Line I K, is in that Plane, becauſe it is
in the Line I K ; and the Line O V is in the ſame Plane, be-
cauſe it is parallel to the Original Line I K, (by *Def.* 5.)
Wherefore the Line V C is the Interſection of the Plane I O K
with the Picture, and conſequently *i k*, which is the Repre-
ſentation of the Line I K, is Part of the Line V C, which paſſes
thro' the Vaniſhing Point V, and the Interſection C, of the
Original Line I K.

COROL. 1.

Hence the Repreſentations of any Number of Lines that
are parallel to each other, but not parallel to the Picture,
will paſs thro' the ſame Point. For they all paſs thro the ſame
Vaniſhing Point. (By this *Theor.* and by *Cor.* 1. *Def.* 5.) See
this expreſs'd in *Fig.* 3.

COROL. 2.

But if the Original Lines are parallel to the Picture, as
well as to each other, their Repreſentations will be parallel
to each other, and to the Originals. For the Line paſſing
thro' the Spectator's Eye, which in other Caſes produces the
Vaniſhing Point by its Interſection with the Picture, is in this
Caſe parallel to it, and therefore produces no Vaniſhing Point.
So that the Repreſentations can never meet each other ; nor
that

that Line paffing thro' the Spectator's Eye, and confequently
are parallel to each other, to that Line, and to the Originals.
See this reprefented in *Fig.* 4. where A B is the Picture, O the
Eye of the Spectator, &c.

COROL. 3.

And hence it appears that the Reprefentations of plain
Figures, parallel to the Picture, are exactly of the fame Shape
as their Originals. For (*Fig.* 5.) the Picture being A B, the
Original Figure parallel to it being C D E F, and the Repre-
fentation being *c d e f,* if you refolve the Original Figure into
Triangles, by means of Diagonal Lines, fuch as D F, the Re-
prefentation will be refolved into Triangles by correfponding
Diagonals *d f.* Whence all the Lines in the Reprefentation
being parallel to all the Lines in the Original, every Triangle
c d f will be like the correfponding Triangle E D F, and con-
fequently all the Lines of the Figure *e d c f* will be in the
fame Proportion to one another, as the correfponding Lines
in the Figure C D E F.

PROP. 2. THEOR. 2.

*Any Line in the Reprefentation of a Figure parallel
to the Picture, is to its Original Line, as the
Principal Diftance is to the Diftance between the
Spectator's Eye, and the Plane of the Original
Figure.*

Let O G (*Fig.* 5.) be perpendicular to the Original Plane,
and to the Picture, cutting them in G, and *g*. The Original
Plane being parallel to the Picture, therefore all the vifual
Rays O C, O D, O E, &c. are cut in the fame Proportion
by the Points *c d e,* &c. as O G is by the Point *g* ; and *d c*
being parallel to D C, the Triangle *d* O *c* is like the Trian-
gle D O C; wherefore *d c* is to D C, as O *d* is to O D, that
is as O *g* (which is the principal Diftance) is to O G.

PROP.

Prop. 3. Theor. 3.

The Representation of a Line is parallel to a Line passing thro' its Directing Point and the Spectator's Eye.

For the Directing Point G, (*Fig.* 2.) and consequently the Line O G, is in the Plane I O K, which produces the Representation *i k*. (By *Prop.* 1.) Wherefore the Directing Plane being parallel to the Picture, (by *Def.* 9.) the Representation *i k* is parallel to O G.

COROL.

Hence it appears that Original Lines which pass thro' the same Point of the Directing Plane, have parallel Representations. Which also is true, when they don't pass thro' the same Point of the Directing Plane, but thro' any Points of the same Line (O G) that passes thro' the Spectator's Eye.

Prop. 4. Theor. 4.

The Distance between the Vanishing Point of a Line and the Representation of any Point in it, is to the Distance between the Vanishing Point and Intersection, as the Distance between the Directing Point and the Intersection, is to the Distance between the Directing Point and the Original Point.

(*Fig.* 2.) Every thing remaining as is explained in the Definitions, and *k* being the Representation of K, the Line V C being Parallel to O G, the Triangles O V *k*, and K G O are similar; wherefore V *k* : O G :: O V : G K. But O G = V C, and O V = G C; wherefore V *k* : V C :: G C : G K.

C

C O R O L.

If you imagin a Plane to pass through K parallel to the Picture, this Proportion will be the same as in *Prop.* 2.

Section II.

Propositions relating to the General Practice of Perspective.

P R O P. 5.   P R O B. 1.

Having given the Center and Distance of the Picture, to find the Representation of a Point, whose Seat on the Picture, and Distance from it are given.

Fig. 6.   S is the Center of the Picture, and C the Seat of the Original Point. Thro' S and C draw at pleasure two parallel Lines, and make S O equal to the Distance of the Picture, and C P equal to the Distance of the original Point from it's Seat, and draw O P, which will cut S C in *p*, which is the Point sought.

When the Picture lies between the Spectator's Eye, and the original Point, *p* will fall between S and C ; but when the original Point falls between the Spectator's Eye and the Picture, then will *p* fall beyond C, as is expressed by the Letters P1 and *p*1; but when the Spectator's Eye falls between the Picture and the original Point, that Point cannot be seen on the Picture, because it is behind the Spectator, though it's Projection is of use in some Cases.

D E M O N-

DEMONSTRATION.

For you may imagin O to be the Spectator's Eye, and P to be the original Point, in their proper Places, then will O P be a visual Ray projecting the Point p on the Picture.

The Point p may be found likewise by Calculation, for $Sp : SC :: OS : OS + CP$, or $Sp_1 : SC :: OS : OS - P_1C$.

PROP. 6. PROB. 2.

*Having given the Center and Distance of the Pi-
cture, and the Position of an Original Line with
respect to the Picture, to find its Indefinite Re-
presentation and Vanishing Point.*

I. *By the Seat, Intersection, and Inclination of the Original Line.*

Fig. 7. Let S be the Center of the Picture, C the Inter-
section of the original Line, and C D its Seat on the Picture.

Draw C A, so that the Angle A C D may be equal to the
Angle the original Line makes with its Seat C D. Draw
S V parallel to C D, and perpendicular to it draw S O equal
to the Distance of the Picture, and draw O V parallel to
C A cutting S V in V, and draw C V. Then will V be
the Vanishing Point, and C V the indefinite Representation
of the Original Line.

II. *By the Seats of two Points of the Original Line.*

Let D and E (*Fig.* 7.) be the Seats of two Points of the
Original Line on the Picture. Draw D E, and S V
parallel to it. Perpendicular to D E take D A, and E B
equal to the Distances of the two Original Points from
their

their Seats; and Perpendicular to S V take SO equal to
the Diſtance of the Picture, and draw O V parallel to
A B, cutting S V in V. Then will V be the Vaniſh-
ing Point. Draw S E, and find the Repreſentation of
B. (By *Prop.* 5.) Then will V *b* be the indefinite Repreſen-
tation. Or find the Repreſentations *a* and *b* of the two Points
A and B (by *Prop.* 5.) and draw *b a* cutting S V in V.
Then will V *b* be the indefinite Repreſentation, and V the
Vaniſhing Point.

The Vaniſhing Point V, may likewiſe be found by a Line
of Tangents; for the Angle V O S is equal to C A D which
is the Compliment of the given Angle A C D; wherefore
the Diſtance of the Picture S O being Radius, S V will be
the Co-tangent of the Angle A C D.

DEMONSTRATION.

Suppoſe the Triangles S V O and C D A to be turn'd round
the Lines S V, and CD, till O co-incides with the Spectator's
Eye, and C A co-incides with the Original Line. Then the
Planes S O V, and C A D being parallel to each other, and
the Angles O V S, and A C D ſtill equal, O V will be pa-
rallel to C A. Wherefore V will be the Vaniſhing Point,
(by *Def.* 5.) and C V will be the indefinite Repreſentation.
(By *Prop.* 1.)

PROP. 7. PROB. 3.

Having given the Center and Diſtance of the Pi-
cture, and the Poſition of an Original Plane, to
find its Vaniſhing Line, its Center, and Diſtance.

Find the Vaniſhing Points of two Lines in that Plain, (by
Prop. 6.) and a Line paſſing through them will be the Vaniſh-
ing Line ſought. (By *Cor.* 2. *Def.* 6.) Or let S be the Cen-
ter of the Picture, (*Fig.* 8.) and A B the Interſection of the
Original Plane. Draw SO parallel to A B, and equal to the

<div align="right">Diſtance</div>

Diftance of the Picture, and draw S V perpendicular to it, and let C be the Angle of the Inclination of the Original Plane to the Picture. Draw O V cutting S V in V, fo that the Angle O V S may be equal to C, and draw V D parallel to A B. Then will V D be the Vanifhing Line fought, V its Center, and V O its Diftance.

The Diftance S V may alfo be found by a Line of Tangents ; for S O being Radius, S V is the Co-tangent of C.

DEMONSTRATION.

Turn the Triangle S V O round the Line S V, till its Plane becomes perpendicular to the Picture. Then will the Plane O V D be parallel to the Original Plane, and confequently will produce the Vanifhing Line D V. (By *Def.* 6.) And O V being perpendicular to V D, V will be its Center, and V D its Diftance, (by *Def.* 7. and 8.)

PROP. 8. PROB. 4.

Having given the Center and Diftance of the Picture, and the Vanifhing Line of a Plane, to find the Vanifhing Point of Lines perpendicular to that Plane.

Fig. 9. S is the Center of the Picture, and V A the Vanifhing Line given. Draw S O parallel to V A, and equal to the Diftance of the Picture, and draw S V perpendicular to V A cutting it in V. Then draw V O, and O P perpendicular to it, which will cut V S in the Point fought P.

Or you may find the Diftance S P by Calculation, for V S : S O :: S O : O P.

DEMON-

DEMONSTRATION.

If the Plane V O P be turn'd round the Line V P, till O co-incides with the Spectator's Eye ; then will O V A be the Plane producing the Vanishing Line V A, and O P being perpendicular to that Plane, will be parallel to the Original Lines that are perpendicular the Original Planes, and therefore will produce their Vanishing Point P.

N. B. *When the Vanishing Line passes thro' the Center of the Picture, the Points S and V will be all one, and the Distance S P, or V P will be Infinite, and the Representations of all Lines perpendicular to the Original Plane, will be perpendicular to the Vanishing Line, and consequently parallel to each other, as they are to the Originals.*

<div align="center">

P R O P. 9.   P R O B. 5.

</div>

Having given the Center and Distance of the Picture, and the Vanishing Point of a Line ; to find the Vanishing Line of Planes perpendicular to that Line.

Fig. 9.   S is the Center of the Picture, and P the Vanishing Point given.   Draw S P, and perpendicular to it take S O equal to the Distance of the Picture, and draw O P, and perpendicular to it draw O K cutting P S in V.   Then draw V A perpendicular to V P, and that will be the Vanishing Line sought, V its Center, and V O its Distance.

This is demonstrated as the foregoing *Prop.* by bringing O to the Spectator's Eye.   And S V may in the same manner be found by Calculation.

<div align="right">

N. B. *When*

</div>

N. B. *When the Point P co-incides with the Center of the Pi-*
&ture, the Distance P V, or S V will be Infinite, and there
will be no Vanishing Line, the Original Planes being paral-
lel to the Picture.

Prop. 10. Prob. 6.

Having given the Center and Distance of the Picture,
and the Vanishing Line of a Plane, and the Va-
nishing Point of the Interfection of that Plane,
with another Plane perpendicular to it ; to find the
Vanishing Line of that other Plane.

Fig. 9. S is the Center of the Picture, A V the Vanishing
Line given, and in it (by *Cor.* 4. *Def.* 6.) D is the Vanishing
Point given. Find P the Vanishing Point of Lines perpendi-
cular to the Plane, whose Vanishing Line is DA. (By *Prop.*
8.) Then draw D P, which is the Vanishing Line sought.

DEMONSTRATION.

For the Plane whose Vanishing Line is sought being per-
pendicular to the other Plain, P will be the Vanishing Point
of one Line in it. But D is the Vanishing Point of another
Line in it by the Supposition. Wherefore D P is its Vanishing
Line.

N. B. *When the Vanishing Line A D passes through the Center*
of the Picture, the Distance V P will be Infinite, (by Prop.
8.) Wherefore in that Cafe D P will be parallel to V P, and
consequently perpendicular to A D.

PROP. 11. PROB. 7.

*Having given the Center and Distance of the Picture,
and the Inclination of two Planes, and the Vanish-
ing Line of one of them, together with the Va-
nishing Point of their Intersection, to find the Va-
nishing Line of the other Plane.*

Fig. 10. S is the Center of the Picture, A B the given
Vanishing Line of one of the Planes, and A the Vanishing
Point of its Intersection with the other Plane, the Planes be-
ing inclined to each other in the Angle C. Find the Vanish-
ing Line T B of a Plane perpendicular to the Line whose Va-
nishing Point is A, (by *Prop.* 9.) its Center will be V, where
it is cut by A S. Take V O equal to the Distance of the
Vanishing Line B T, and thro' the Point B, where it cuts
the given Vanishing Line A B, draw B O, and make the An-
gle B O T equal to C, the Line O T cutting T B in T, and
draw A T, which will be the Vanishing Line sought.

DEMONSTRATION.

Turn the Triangle B O T round the Line B T, till O co-
incides with the Spectator's Eye; then will O A be perpendi-
cular both to O B and to O T, it being perpendicular to the
Plane B O T by the Construction, and O A will be parallel
to the Intersection of the Original Planes. Wherefore A B
being the Vanishing Line of one of them, and the Angle BOT
being equal to C, T A will be the Vanishing Line of the
other of them. (By *Cor.* 3 and 4. *Def.* 6.)

N. B. *When the Intersection of the Planes is parallel to the Picture,
the Point A being at an infinite Distance, S A will be parallel
to the Vanishing Line B A, and V will co-incide with S, and
B V will be perpendicular to B A; V O will be the Principal
Distance, and T A will be parallel to B A.*

PROP.

PROP. 12. PROB. 8.

Having given the Vanishing Line of a Plane, its Center and Distance, and the Vanishing Point of a Line in that Plane, to find the Vanishing Point of Lines that make a given Angle with that Line.

Q V (*Fig.* 11.) is the Vanishing Line given, S its Center, and V the Vanishing Point given. Perpendicular to Q V draw S O equal to the Distance given. Then draw O V and make the Angle V O Q equal to the given Angle, and Q will be the Vanishing Point sought. So that if you draw any two Lines V P, and Q P, the Angle V P Q will represent an Angle equal to V O Q.

DEMONSTRATION.

Turn V O Q round the Vanishing Line V Q, till O co-incides with the Spectator's Eye. Then will O V and O Q be parallel to the Original Lines whose Vanishing Points are V and Q, and consequently the Angle they make with each other is equal to V O Q.

By this Proposition you may find the Representations of any plain Figures, having one side. For the Original Figure may be resolved into Triangles, whose Angles being all given, the Vanishing Points of all their sides will be given by this Proposition. Whence the Representations of all the sides will be found, beginning first with those that lie next the side given.

Here likewise you may observe, That the Center of the Picture is not concerned in this *Problem*. Wherefore to see the Representations of Figures in a Plane, whose Vanishing Line is V Q, the Spectator's Eye may be placed any where in the Circumference of a Circle described by O, while the Plane V O Q turns round the Axis V Q.

PROP.

PROP. 13. PROB. 9.

Having given the Representation of a Line, and its Vanishing Point; to find the Representation of a Point that divides the Original Line in a given Proportion.

Fig. 12. A B is the Representation given, and V the Vanishing Point. Draw at pleasure A *b*, and divide it in the given Proportion in *c*. Draw V O parallel to A *b*, and draw *b* B cutting it in O. Then draw O *c*, which will cut A B in the Point sought C.

This may likewise be done by Calculation; for $AC : AB :: Ac \times AV : Ac \times AV + bc \times BV$, or $AC_1 : AB :: Ac_1 \times AV : Ac_1 \times AV - bc_1 \times BV$, or $AC_2 : AB :: Ac_2 \times AV : bc_2 \times BV - Ac_2 \times AV$.

DEMONSTRATION.

Suppose the Original Line to be in a Plane, whose Vanishing Line is V O. Then will O A, O B *b*, and O C *c* represent parallel Lines. Wherefore the Originals of A B and A *b* are divided in the same Proportion by the Originals of the Points C and *c*. But A *b* being parallel to V O, it is divided by *c* in the same Proportion as its Original (by *Cor.* 3. *Prop.* 1.) that is, in the Proportion given. Wherefore the Original of A B is also divided in the Proportion given.

PROP. 14. PROB. 10.

Having given the Vanishing Point of a Line V (Fig.
13.) and three Points in its Representation, A,
B, C; to find a fourth Point D, so that the Part
represented by C D may be to the Part represented
by A B, in a given Proportion.

Draw at pleasure V O, and A *d* parallel to each other, and
thro' any Point O of the Line V O draw V B, V C cutting
A *d* in *b*, and *c*. Make *c d* to A *b* in the given Proportion,
and draw O *d* cutting A V in D, which will be the Point
sought.

This is demonstrated as the foregoing *Proposition.*

The Point D may also be found by a Scale and Compasses,
by the help of this Proportion C D : C V :: *c d* × A B ×
C V : *c d* × A B × C V + A *b* × A V × B V. Which may
easily be accommodated to any Position of the Points A, B,
C, D, only observing well the Signs + and —.

PROP. 15. PROB. 11.

Having given the Vanishing Line of a Plane, its
Center and Distance, and the Representations of
two Lines in that Plane, from a given Point in one
of them to cut off a Line, that shall represent a
Line in a given Proportion to the Line represented
by the other.

Fig. 14. S D is the given Vanishing Line, and V its
Center, and A B and C I are the Representations given, whose
Vanishing Points are D and E. It is required to find the
Point I, so that the Line represented by C I, may be to the Line
represented by A B, as *c* is to *d*. Draw V O perpendicular to

F D, and equal to the Distance given, and draw O D and O E, and make O Q : O R : : *c* : *d*, and draw O S parallel to R Q. Draw A C cutting S D in F, and draw F B and D C cutting each other in *p*. Then draw *p* S, which will cut C E in the Point sought I.

DEMONSTRATION.

Because of the Vanishing Points D and F, the Figure A B *p* C represents a Parallelogram. Wherefore C *p* represents a Line equal to the Line represented by A B.

By the manner of placing the Point O, the Plane S O D may be considered as the Plane producing the Vanishing Line S D, O being the Spectator's Eye. Wherefore O S being parallel to R Q, D, E and S are the Vanishing Points of the Sides of a Triangle parallel to R O Q. Wherefore *p* C I represents that Triangle, and consequently the Line represented by C I is to the Line represented by *p* C, or by A B, as O Q is to O R; that is, as *c* is to *d*.

If the Point F falls out of reach, you may bring it into the bounds of the Picture thus. Instead of A B suppose *a b* to be one of the Lines given, whose Vanishing Point is D. Draw D A, and taking any Point *r* in the Vanishing Point F D, as is most convenient, draw *r a* and *r b* cutting D A in A and B, and use the Line A B, as above, instead of *a b*. For because of the Vanishing Points *r* and D, A B and *a b* represent equal Lines.

Prop. 16. Prob. 12.

Having given the Vanishing Line of a Plane, and the Representations of two Lines in it ; to find a Point in one of them , so that it may represent a Line divided in the same Proportion, as the Line represented by the other.

Fig. 15. P *k* is the Vanishing Line given, and A C, D E are the two Representations, and it is required to find the Point F, so that the Parts represented by D F and F E may be in the same Proportion as the Parts represented by A B and B C. Draw any Line *s v* parallel to the Vanishing Line, and by means of any Vanishing Point *w* transfer the Points A, B, C, to *s*, *t*, *v*, and *s v* will be divided in the same Proportion as the Original of A C (by *Cor.* 3. *Prop.* 1.) and because of *w s*, *w t*, *w v*, representing Parallels. Whence you may find the Point F by *Prop.* 13.

When the Vanishing Point *k* of the Line A B falls within reach, you may find the Proportion of *s t*, to *t v*, without drawing any Lines on the Picture. For *s t* : *t v* :: A B × C *k* : B C × A *k*.

By these Propositions you may find the Perspective Representation of any Figures proposed. For you may consider the whole Design as one Figure, the Position of whose Parts are all given with respect to each other. Then having found the Vanishing Line of some Principal Plane, and the Representation of a Remarkable Line in the Design, by *Prop.* 5, 6, 7, you may find the Vanishing Lines of all other Plains, by their Positions with respect to the Plane first assumed, by *Prop.* 9, 10, 11. And then by the Vanishing Points already given, and by the given Proportions of the Parts, you may find the Representations of the Figures proposed by *Prop.* 12, 13, 14, 15, 16. Or having given the Plan of any Figure, upon any Plane that is conveniently chosen, and the Elevations of the Parts of the Figure above that Plane, having found the Representation of the Plan, by *Prop.* 12, or by that and *Prop.* 13, 14, 15, 16, you may then find the Representations of the Elevations, by the help of *Prop.* 8. And in order to do all this, there is no necessity of having an Original Design drawn out in its just Proportions ; but it is sufficient to have the Proportions of the Parts express'd in Numbers; the Design being any how scetch'd out to help the Memory. This I shall illustrate by a few *Examples.*

EXAMPLE I.

To find the Reprefentations of any Figures on a Plane, having given the Interfection, and Vanifhing Line, Center and Diftance, the Original Plane being drawn out in its juft Proportions.

At the end of *Prop.* 12. I have obferved that the Shapes of the Reprefentations of Figures on a Plane don't at all depend upon the Angle the Picture makes with that Plane. Wherefore let F E (*Fig.* 16,) be the Interfection given, *f e* the Vanifhing Line, V its Center, and V O its Diftance perpendicular to it. Then O being confider'd as the Spectator's Eye in the Plane X, which generates the Vanifhing Line, and which is now fo turn'd as to co-incide with the Plane of the Picture Y, let Z be the Original Plane turned in the fame manner till it co-incides with the Picture, the Figures A B C D and M being now feen on the back-fide. Then to find the Reprefentation of any Point B, draw B E cutting E F in E, and draw O *e* parallel to it, cutting the Vanifhing Line in *e*; then E being the Interfection, and *e* the Vanifhing Point of the Line B E, the Reprefentation of the Point B is found by drawing E *e*, and the vifual Ray O B cutting it in B. The Point C is found without drawing the vifual Ray, by the Interfection of the indefinite Reprefentations E *e*, and F *f* of the Lines C E and C F. And in the fame manner the Reprefentation *m* of the Figure M, is found by the Interfections and Vanifhing Points, without drawing any vifual Rays.

Or you may find the Reprefentations of the Figures propofed by means of the Directing Plane. Thus (*Fig.* 17.) D F C being the Original Plane reverfed, as before, D F G H the Plane of the Picture, D F the Interfection, G H the Vanifhing Line, V its Center, and V O its Diftance fet off perpendicular to it, as before; having drawn *d f* parallel to G H, and as far below O, as D F is below G H, *d* O F will be the Directing Plane brought into the Plane of the Picture, *d f* being its Interfection with the Original Plane. Whence having
continued

continued the Sides of the Triangle A B C, till they cut D T and *d f* in D, E, F, and *d*, *e*, *f*; D, E and F being their Interfections, and *d*, *e*, *f*, their Directing Points, drawing D *b* parallel to O *d*, E *c* parallel to O *e*, and F *c* parallel to O *f*, they by their Interfections will give the Reprefentation fought *a b c*, (by *Prop.* 3.)

EXAMPLE II.

To find the Reprefentation of any given plane Figure, having given the Reprefentation of one Side of it, and the Vanifhing Line, Center and Diftance of the Plane it is in. (*Fig.* 18.) M F is the given Vanifhing Line, V its Center, and V O its Diftance fet off perpendicular to it, and *a b* is the Line given to reprefent the Side A B of the Figure A B C D E. Having refolved the Original Figure into Triangles, by means of the Diagonals A C, A D, the Vanifhing Points of the Sides A C, B C, are found by continuing *a b* till it cuts the Vanifhing Line in its Vanifhing Point F; and then making the Angle F O H equal to B A C, you have the Vanifhing Point H of the Side A C; and by making the Angle F O G equal to *p* B C, you have the Vanifhing Point G of the Side B C: Whence you have the Reprefentation *a b c* of the Triangle A B C. And in the fame manner by the Side *a c* you will get the Triangle *a c d*, and then by *a d* you get the Triangle *a d e*, and by that means have the Reprefentation *a b c d e* of the whole Figure. This is founded upon *Prop.* 12.

Or you may proceed thus. Continue the Sides D C, D E (*Fig.* 19.) till they cut A B in P and Q. Then having found the Reprefentations *p* and *q* of the Points P and Q (by *Prop.* 13.) you will find the Reprefentation of the Triangle P D Q, as before (by *Prop.* 12.) and then you will have the Points *c* and *e*, by *Prop.* 13.

Or having continued *a b* (*Fig.* 20.) till it cuts the Vanifhing Line in F, and drawn D E parallel to the Vanifhing Line H F, cutting *a b* in D, draw D B parallel to O F, and draw the vifual Rays O *a*, O *b*, cutting D B in A and B, and on the

the Side A B make a Figure A C B like the Original Figure, and proceed as in *Exam.* 1. the Line D E being confidered as the Interfection, and D C E as the Original Plane inverted.

Curve-lined Figures are defcribed by finding feveral Points, and then joyning them neatly by Hand.   And this may conveniently be done by putting the Geometrical Defcriptions of Curves into Perfpective, as in the two following Examples of defcribing a Circle.

EXAMPLE III.

Having given the Vanifhing Line of a Plane, its Center and Diftance ; to find the Reprefentation of a Circle, from the given Reprefentation of one Radius.

Fig. 21. C is the Reprefentation of the Center, and C A of the Radius given, D E is the Vanifhing Line, and O the Spectator's Eye placed as in the foregoing Examples. Draw at Pleafure C B, and make C B to reprefent a Line equal to that reprefented by C A (by *Prop.* 15.) that is, bifect the Angle E O D by the Line O F, and draw F A, cutting E C in B.   And in the fame manner you may find as many Points B in the Circumference of the Circle, as you pleafe.

EXAMPLE. IV.

Having given the Vanifhing Line of a Plane, its Center and Diftance, thro' three Points given to draw the Reprefentation of a Circle.

Fig. 22. A, B, and C are the three Points given, D*f* the Vanifhing Line, and O the Spectator's Eye placed as above. Draw C A and C B cutting the Vanifhing Line in D and E, and draw D O and E O.   Draw any Line *d* O, and make the Angle *d* O *e* equal to D O E, or having made an Inftrument containing the Angle D O E, turn it round the Center O, till it comes into the Pofition *d* O *e*, the Leg O D cutting
the

the Vanishing Line in *d*, and O E, (or O *e*) cutting it in *f.*
Thro' *d* and *f* draw *d* A and *f* B cutting each other in *p*, and
you will have one Point in the Representation fought. This
Construction is taken from the equality of Angles in the Circle
insisting upon the same Base, whence the Vanishing Points are
found by *Prop.* 13.

E X A M P L E V.

Having given the same things as in either of the two fore-
going Examples, from a given Point of the Picture to draw a
Line that shall touch the Representation of the Circle.

Fig. 23. D E is the Vanishing Line, O the Eye of the Spe-
ctator, C the Representation of the Center, and C A the Re-
presentation of the Radius, and P the Point given, from
whence a Tangent is to be drawn. Draw P C cutting the
Vanishing Line in E, and (by *Exam.* 3.) find the Radius
C B. Any where apart draw *b p*, and make *b c* : *c p* : : B C
× P E : C P × B E; that is, make the line *b c p* like the O-
riginal of B C P. With the Center *c* and Radius *c b* make a
Circle, and draw the Tangent *p* T, and then find the Repre-
sentation P *t* of it by *Exam.* 2. And you may proceed in the
same manner by finding the Original Figure, when three Points
are given.

If instead of from the Point P, it had been required to draw
a Tangent parallel to C E, the Point P being supposed infi-
nitely distant, P E would be equal to P C, and consequently
it would be *b c* : *c p* : : B C : B E. But in this Case, as also
always when P falls beyond E, you must take *c p* backwards,
p falling on the contrary side of the Point *c*, to what it does in
the present Scheme. But I shall leave the Reader to consider
all the variety of Cases that may happen from the different Po-
sition of the Points.

The Use of this Example is to find the apparent part of the
Base of a Cone or Cylinder.

E X A M-

EXAMPLE VI.

Fig. 24. In this Figure having given the Center of the Picture S, its Distance equal to the Line L, and the Line A B to represent one side of the first Step, and K I the Vanishing Line of the Plane of the Horizon; the Representation of the Base A B C D of the first Step, is found by the Vanishing Points I, K, L, which are found by *Prop.* 12. Then by *Prop.* 8. having found the Vanishing Point M of the Perpendiculars A E, F C, &c. the Vanishing Line of the Face A C E F being M K, the Vanishing Point N of the Diagonal A F is found by *Prop.* 12. Whence all the rest of the Lines are found, as appears sufficiently in the Figure.

EXAMPLE VII.

Fig. 25. In this Figure the Representation H I K Q of a regular Tetraedrum is found as follows; Having given the Side H I, the Center of the Picture S, its Distance equal to the Line L, and G F the Vanishing Line of the Face H I K. Having continued the given Side H I till it cuts the Vanishing Line G F, in its own Vanishing Point F, the Vanishing Points G and M of the Sides I K and H K are found by *Prop.* 12. Then having made an Equilateral Triangle A B C, and drawn A D perpendicular to B C, and made the Isosceles Triangle A E C, whose Sides A E, and C E are equal to A D; the Angle A E C being equal to the Inclination of two Faces of the Figure proposed, the Vanishing Line F N of the Face H I Q is found by *Prop.* 11. Whence having the Vanishing Points P and N of the Sides H Q and I Q, you have the whole Representation sought.

The Reader may exercise himself in drawing the Representations of the five regular Solids, in order to which their Plans and Profils may be found in the following manner.

Fig. 26. Having made an Equilateral Triangle A B C, and found its Center G, you have the Plan A B C G of a regular

Tetraedron.

Tetraedron. Then making the Triangle B C E, whose Sides B E, and C E are equal to the Perpendicular B D, and drawing C F perpendicular to B E, you have C F equal to the height of the Vertex above the Center G.

It would be superfluous to shew how to make the Plan and elevation of a Cube.

Fig. 27. To find the Plan and elevation of a regular Octaedron, make the regular Hexagon A E B F C D, and having drawn the Lines, as appears in the Figure, you have the Plan of the Figure, on the Plane of one Face A B C. Then having drawn B D cutting A C in G, and D H perpendicular to it, with the Center G and Radius G B make a Circle cutting DH. in H, and D H will be the Distance between two opposite Faces.

Fig. 28. To find the Plan of a regular Dodecaedron, make a regular Decagon A B C D E F G H I K, whose Center is S. Then draw a Diagonal A D, leaving out two Angles B and C, and draw S B cutting it in M, and make two regular Pentagons M N O P L, *m n o p l* in sub-contrary Positions, having the same Center S, and draw the Lines N D, O F, P H, L K, *o* A, *p* C, &c. and you will have the Plan of a Dodecaedron on one of its Faces M N O P L.

Draw *a v*, and in it make *a b* and *b v* equal to the Perpendicular L R let fall from one Angle of the Pentagon to the opposite Side, and make the Isosceles Triangle *c b v*, with the Base *c v* equal to B D, or, which is all one, to a Diagonal L N of the Pentagon. In *v c* take *c e* equal to L M, and make *a d, d f, e f* parallel and equal to their opposite Lines *c e, c b, b a,* and let *s* be the Center of the Figure *a b c e f d.* Then bisect *c v* in *g*, and draw *s g* cutting *b c* in *h,* and make *n m* perpendicular to *a d, s m* and *s n* being each equal to half L M, and make *b i, f k, f l* each equal to *b h,* and having drawn the Lines as appears in the Figure, you will have the Profil of the same Figure ; *d, n* and *h* being the height of the Angles perpendicular over K, B and H, D and F, and *k, m,* and *o*

E 2   being

being the Elevations of the Angles perpendicular over A and I, C and G, and E, and *f l e* being the Elevations of the upper Face, which is perpendicular over *n o p l m.*

Fig. 29. To find the Plan and Profil of a regular Icosaedron, make a Right-Angle A B C, B C being the double of A B, and with the Center A and Radius A C make a Circle cutting A B in D. Then make a regular Hexagon E F G H I K, whose Center is S, and having drawn the Lines S E, S F, S G, &c. make the Distances S L, S M, &c. to S F, as B D is to B C, that is, as the Side of a Pentagon is to the Diagonal, and draw all the Lines as appears in the Figure, and you will have the Plan of a Icosaedron on one of its Faces L M N.

To find the Profil of the same Figure make the Isosceles Triangle O P Q, whose Base O Q is equal to the Difference between L M and E G, and the Sides O P, Q P are equal to a Perpendicular let fall from L to the opposite Side M N of the Triangle L M N. In Q P make P R = P O, and in Q O make O T = L M, and make R X, X V, V T parallel and equal to their opposite Lines O T, O P, P R, and draw the Lines as appears in the Figure, and you will have the Profil sought, Z being the Elevation of the Points over F and K, Y that over G and I, R that over H, T that over E, V that over *m* and *n*, and X that over *l* ; and observe that the Lines Z R, and T Y are parallel to O P, or V X.

EXAMPLE VIII.

Fig. 30. Having given the Center of the Picture S, its Distance S D, and the Representation C A of one Radius of a Sphere, whose Vanishing Point is V, the Representation of the Center being C, the Representation of the Sphere is found as follows. Draw S R perpendicular to A V, and in it take R O equal to the Hypothenuse of a Right-angled Triangle, whose Base is S R, and perpendicular is S D, that is, make R O equal to the Distance of the Spectator's Eye from the Point R. Draw A O, V O, C O, and draw any Line parallel to V O,

cutting

cutting A O, and C O in *a* and *c*, and with the Center *c*, and Radius *c a* describe a Circle, to which draw a Tangent O *t* cutting C A in T. Then find the Vanishing Line P Q of a Plane perpendicular to the Line, whose Vanishing Point is C (by *Prop.* 9.) and find the Representation of a Circle in that Plane, the Representation of one Radius being C T, and the Center C, (by *Exam.* 3.) and that will be the Contour of the Sphere.

The Demonstration of this Construction is easy to one that considers, That the visual Rays which touch a Sphere make a Cone, whose Base is a lesser Circle of the Sphere, whose Representation is the Contour of the Representation of the Sphere.

In the Practice of *Perspective* it is often Necessary to draw Lines towards a Vanishing Point, or some other Point that is out of reach, in which case the following Propositions are useful.

P R O P. 17. P R O B. 13.

From a given Point to draw a Line that shall tend to an inaccessible Point of a given Line, having the Distance of the inaccessible Point from a given Point of that Line.

Fig. 31. Q is the inaccessible Point in the Line P Q, the Distance B Q being given, and A is the Point from whence a Line is to be drawn tending towards Q.

Draw at pleasure A P, cutting P Q in P, and taking the Measure of P B by a Scale of equal Parts, you will have the length of P Q. Take any Point *q* in the Line B Q, and make P *a* : P *q* : : P A : P Q. Draw *a q*, and A Q will be parallel to it.

P R O P.

Prop. 18. Prob. 14.

*Having given two Lines that tend to an inacceffible
Point, thro' a given Point to draw a Line that
fhall tend to the fame Point.*

Fig. 32. A B and C D are the given Lines tending to the
inacceffible Point, and P is the Point thro' which it is requi-
red to draw a Line P *p* tending to the fame Point.

Draw at pleafure P A (*n* : 1) and P B parallel to it, cut-
ting A B and C D in C, A, B, and D, and make B *p* : B D
: : A P : A C, and draw P *p*, which will be the Line fought.

Otherwife, without the Compaffes.

n : 2, and *n* : 3. Thro' P draw at pleafure two Lines cut-
ting A B and C D in A and C, B and D. Then draw A D
and B C meeting in Q. Thro' Q draw at pleafure two Lines
cutting A B and C D in *b* and *c*, *a* and *d*, and draw *a c*
and *b d* meeting in *p*, and draw P *p*, which will be the Line
fought.

Section III.

Of finding the Shadows of given Figures.

In this Place I confider Shadows only as the Projections of
given Figures on given Surfaces, by the means of given Lumi-
nous Points. For I confider the Luminous Body as a Point,
to avoid the Difficulties that would attend the Defcription of
Shadows, if the Magnitude of the Luminous Body were taken
into Confideration, it being fufficient for Practice to regard
only the Center of the Luminous Body; and having found the
Contour of the Shadow in that Cafe, the Penumbra may be
drawn

drawn by a good Judgment founded on much Obfervation; it being difficult to bring every thing to exact Mathematical Conftructions, at leaft fo as to be moft convenient for Practice. But the Artift that has made himfelf well acquainted with the Principles of this Art, will eafily find ways to fatisfy himfelf in thofe cafes that require more Exactnefs.

<center>Prop. 19. Prob. 15.</center>

Having given the Center and Diftance of the Picture, and the Vanifhing Line of a Plane, and the Reprefentation of a Luminous Point, and of its Seat on that Plane; to find the Reprefentation of the Shadow of a Point, whofe Reprefentation and Seat on the fame Plane are alfo given; and to find the Vanifhing Point of the Ray of Light.

Fig. 33. E T is the given Vanifhing Line, S the Reprefentation of the Luminous Point, R the Reprefentation of its Seat, A the Reprefentation of the Point whofe Shadow is to be found, B the Reprefentation of its Seat, and P the Vanifhing Point of Lines perpendicular to the given Plane, (found by *Prop.* 8.) Draw R C cutting the Vanifhing Line in V, and draw V P; then draw S A cutting R V in C, and P V in D. Then will C be the Shadow fought, and D the Vanifhing Point of the Ray of Light.

<center>DEMONSTRATION.</center>

For the Originals of R, B, A, and S are in a plain Perpendicular to the Plane, on which the Shadow is to be caft; wherefore C is the Shadow, it being the Interfection of the Ray S A with the Plane R B. Befides, V is the Vanifhing Point of R B. Wherefore P V is the Vanifhing Line of R S A B, and confequently D is the Vanifhing Point of S A.

<div align="right">N. B. *When*</div>

N. B. *When the Original Luminous Point is behind the Specta-*
tor, so that it cannot have any real Representation on the
Picture, its imaginary Representation (which is as it were
the Shadow of the Spectator's Eye on the Picture,) must be on
the contrary Side of the Plane to the Point, whose Shadow is
sought, that is, if B be between P and A, S on the contrary
must be between P and R (as in n : 2.*) And when the Lu-*
minous Point is the Sun, or any other Light supposed to be
at an infinite Distance, the Points R and V will co-incide,
and D will be the same as S, as I have shewn in n : 3, *and*
n : 4. *In* n : 3, *the Sun appears in the Picture: But in*
n : 4, *it is behind the Spectator, and its imaginary Repre-*
sentation is S.

PROP. 20. PROB. 16.

Having given the Center and Distance of the Picture,
and the Representation of a Line, and of the Seat
of one Point of it on a Plane whose Vanishing Line
is given, and also the Vanishing Point of that
Line; to find the Representation of the whole Seat
of that Line on that Plane.

Fig. 34. V E is the given Vanishing Line of the Original
Plane, P is the Vanishing Point of Lines perpendicular to it,
(found by *Prop.* 8.) A C is the given Representation of a Line,
D its Vanishing Point, and B the Representation of the Seat
of one Point of it, whose Representation is A. Draw P D
cutting V E in E, and V B cutting A D in C. Then will
V B be the Representation of the whole Seat of the Line A D.

DEMONSTRATION.

For the Original of A B C is a Triangle whose Va-
nishing Line is P D. Wherefore V must necessarily be the
Vanishing Point of B C, &c.

When

When you have got the Description of any Figure by the foregoing *Propositions*, you generally have the Vanishing Points of all the Lines of it. So that having the Seat of one Point of the Figure on the Plane you intend to cast the Shadow on, you may find the Seats of all the other Points of the Figure by this *Proposition*, and their Shadows by the foregoing.

<center>P R O P. 21. P R O B. 17.</center>

Having given the Center and Distance of the Pi-
cture, and the Vanishing Lines of two Planes,
and the Representation of their common Interse-
ction, and the Representation of a Line, and of
its Seat on one Plane; to find the Representation
of the Seat of that Line on the other Plane.

Fig. 35. A B and C B are the Vanishing Lines of the two Planes, B D the Representation of their common Intersection, F E the given Representation of a Line, and A E the Representation of its Seat on the Plane, whose Vanishing Line is A B. By *Prop.* 8. find the Vanishing Points G and H of Lines perpendicular to the Planes, whose Vanishing Lines are A B and C B. Draw A G cutting F E in F, and B C in C, and draw H F cutting B C in I, and draw C D cutting E F in K. Then draw I K, which will be the Line sought.

<center>D E M O N S T R A T I O N.</center>

F, A, and C are the Vanishing Points of the Lines E F, E A, and K C, whose Originals are all in the same Plane, because of the Vanishing Line G C, and because they meet in E and D. But because of the Point D being in the Representation of the Intersection of the two Planes, and because of the Vanishing Line B C, the Original of D K, and conse-quently of the Point K, is in the Plane, whose Vanishing Line is B C. But because of the Vanishing Line H F, I is the

<center>F Vanishing</center>

Vanishing Point of the Seat of E F on the Plane, whose Vanishing Line is B C; wherefore K being the Representation of the Intersection of that Line with that Plane, K I will be the Representation of its Seat.

By this *Proposition* you may carry a Shadow from one Plane to another. For E F being the Ray of Light, having its Position with respect to the Plane, whose Vanishing Line is A B, you find the Shadow K on the Plane, whose Vanishing Line is B C. So that by these three last *Propositions* you may find the Shadows of any Figures.

In some Cases perhaps it may be most convenient to find the Shadows by putting the foregoing Rules of *Perspective* into *Perspective.* For Example (*Fig.* 36.) G, E, and F, being the Vanishing Points of the Sides of the Triangle ABC, whose Shadow is to be cast on a Plane, whose Vanishing Line is T V, and Intersection with the Plane of the Triangle A B C is D I; to find the Shadow cast by the Luminous Point represented by S. I seek the Representation D *g* of the Intersection of a Plane passing through the Luminous Point parallel to the Plane of the Original Triangle, with the other Plane: Then drawing S G, S E, and S F cutting D *g* in *g*, *e* and *f*, and continuing the Sides of the Triangle, till they cut D I in H, I, and K; I consider S as if it were a Spectator's Eye, and use *g*, *e* and *f* as Vanishing Points, and K, H, and I as Intersections of the Lines B C, B A, A C, and so find the Shadow *a b c*.

SECT.

Section IV.

Of finding the Representations of the Reflections of Figures on polish'd Planes.

It is well known that the Reflections of Figures on a polish'd Plane, as in a Looking-Glass, or on the Surface of standing Water, appear to be just as much on one side of the Plane, as the real Objects are on the other side. So that to find the reflected Appearance of any Point, you must draw a Perpendicular to the reflecting Plane from the real Point, and in it take a Point at the same Distance on the contrary side of the Plane. For Instance, To find the apparent Place of the Reflection of the Point P (*Fig.* 37.) on the Plane A B draw the Perpendicular P B, and on the contrary side of the Plane make B *p* = B P, and *p* will be the apparent Place of the Point P, seen by Reflection. And from hence it follows that the Appearance of any Figure seen by Reflection, is exactly of the same Shape, and of the same Bigness as the real Figure, but in an inverted Position.

Upon these Principles depend the following *Propositions*, by which you may easily find the *Perspective* Representations of the Appearances of any Figures seen by Reflexion from given Planes.

PROP.

P R O P. 22.   P R O B. 18.

Having given the Center and Distance of the Picture,
and the Vanishing Line of a reflecting Plane, and
the Representation of the Seat of a Point on that
Plane ; to find the Representation of the Reflecti-
on of that Point.

Fig. 38. A B is the Vanishing Line given, P the Represen-
tation of the real Point, Q the Representation of its Seat on
the reflecting Plane, and V the Vanishing Point of Lines
perpendicular to that Plane.   Make Q *p* to represent a Line
equal to that represented by Q P (by *Prop.* 13.) and *p* will
be the Point sought.

This is evident by the Introduction to this *Section.*

P R O P. 23.   P R O B. 19.

Having given the Center and Distance of the Picture,
and the Vanishing Line of a reflecting Plane, and
the Vanishing Point of a Line, together with the
Representation of its Seat on the reflecting Plane ;
to find the Representation of the Reflection of that
Line, and its Vanishing Point.

Fig. 39. A B is the Vanishing Line of the Reflecting Plane,
V the Vanishing Point of Lines perpendicular to it,   P I the
Representation of the real Line, C its Vanishing Point, and
B I the Representation of its Seat on the Reflecting Plane.
With the Vanishing Point V make B *c*, and B C to represent
equal Lines, (by *Prop.* 13.) then will *c* be the Vanishing
Point of the Reflection ; and drawing *c* I, that will be its
Representation.

DEMON-

DEMONSTRATION.

For if you seek the Reflection of the Line by the Reflections of two Points of it, by the foregoing *Proposition*, then drawing any Line V P, the Parts P Q and Q p will represent equal Lines; otherwise p can't represent the Reflection of P. Wherefore when Q co-incides with B, B C and B c will also represent equal Lines. But in that Case C, B, and c, are the Vanishing Points of the real Line P I, its Seat B I, and its Reflection p I. Wherefore the Vanishing Point c, and the Reflection c I are rightly determined by the foregoing Construction.

PROP. 24. PROB. 20.

Having given the Center and Distance of the Picture, and the Vanishing Line of a reflecting Plane; to find the Vanishing Line of the Reflection of a Plane, whose Vanishing Line is also given.

Fig. 39. A B is the Vanishing Line of the Reflecting Plane, V the Vanishing Point of Lines perpendicular to it, and A C the Vanishing Line of the real Plane. Draw V C cutting A C and A B in C and B, and find the Point c, as in the foregoing *Proposition*, and draw A c, which will be the Vanishing Line sought.

DEMONSTRATION.

For by the foregoing *Proposition* c is the Vanishing Point of the Reflection of a Line, whose Vanishing Point is C. But the Vanishing Line A C is the Place of all the Points C; and therefore the Vanishing Line A c is the Place of all the Points c, and consequently is rightly determined by finding one of them, and drawing A c.

By thefe three *Propositions* having got the Vanifhing Line
of the Reflection of one Face of a given Figure, and the
Reprefentation of the Reflection of one fide of that Face,
you may find the whole Reflection as you do the real Fi-
gure, by the foregoing *Propositions*, only obferving to make
the Reflections in a contrary Pofition to the real Objects.

SECTION V.

Of the Inverfe Practice of PERSPECTIVE,
and of the manner of Examining Pictures al-
ready drawn.

PROP. 25. PROB. 21.

*Having given the Reprefentation of a Line divi-
ded into two Parts in a given Proportion; to
find its Vanifhing Point.*

Fig. 13. Suppofe the Part reprefented by A B, to be to that
reprefented by B C, as *a* to *b*. Draw at pleafure A *d*, and
taking A *b* at pleafure, make A *b* : *b c* : : *a* : *b*, and draw
b B and *c* C meeting in O, and draw O V parallel to A *d*,
cutting A B in V, which is the Vanifhing Point fought.

This is the reverfe of *Prop.* 13.

You may find V by Arithmetick, making C V $=$
$$\frac{a \times AC \times BC}{b \times AB - a \times BC}$$

PROP. 26. PROB. 22.

Having given the Representation of a given kind of Triangle, and its Vanishing Line; to find its Center and Distance.

Fig. 40. A B C is the given Representation of a Triangle, and its Vanishing Line is F E, and consequently the Vanishing Points of its Sides are F, D, and E.

Bisect FD and DE in G and H, and raise the Perpendiculars G I, H K, and make the Angle G I D equal to the Angle that ought to be represented by FBD, and HKE equal to the Angle that ought to be represented by DAE, and with the Centers I, and K, and Radius's I D and K D, make two Circles cutting each other in O. Then draw OP cutting FE at right Angles in P, and P will be the Center, and PO the Distance of the Vanishing Line F E.

DEMONSTRATION.

It is a known Property of the Circle, that the Angles FOD and DOE, are equal to GID, and HKE; wherefore the Angles FBD, or ABC, and BAC, represent Angles equal to GID, and HKE, by *Prop.* 12.

PROP.

Prop. 27.   Prob. 23.

Having given the Representation of a given kind of Parallelogram; to find its Vanishing Line, Center and Distance.

Fig. 41. A B C D is the given Representation of a Parallelogram.   Continue the opposite Sides, till they meet in E and F, then will E F be the Vanishing Line.   Draw the Diagonal A C, and by means of the Triangle A B C find the Center and Distance by the foregoing *Proposition.*

DEMONSTRATION.

For the Original Figure being a Parallelogram, the opposite Sides are parallel ; wherefore their Representations must meet in their Vanishing Points, and consequently E F will be the Vanishing Line.

Prop. 28.   Prob. 24.

*Having given the Representation A B C D (*Fig. 42.*) of a given kind of Trapezium, to find its Vanishing Line, Center and Distance.*

Draw the Diagonals A C and D B meeting in E, and by the given Proportion of the Originals of A E, E C, B E, E D, find the Vanishing Points F and G of the Lines A C and D B (by *Prop.* 25.) and you will have the Vanishing Line G F.   Then by the given Species of the Original of the Triangle A B E, find the Center and Distance (by *Prop.* 26.)

PROP. 29. PROB. 25.

Having given the Representation of a right-angled Parallelepipedon ; to find the Center and Distance of the Picture , and the Species of the Original Figure.

Fig. 43. A B C D E F G is the Representation given. Continue the Sides of the Figure till they meet in their Vanishing Points H, I, K, and you will have the Vanishing Lines H I, I K, K H of all the Faces of the Figure. Then from H and I draw Perpendiculars to I K and H K, meeting in S, which will be the Center of the Picture, L and M being the Centers of the Vanishing Lines I K and H K. Upon the Diameter I K make a Circle cutting H L in O. Then will L O be the Distance of the Vanishing Line I K, Whence the Distance of the Picture is easily found. Draw the Diagonal G F cutting I K in its Vanishing Point N, and draw O N. Then will N O K be the Angle represented by A G F, and N O I will be the Angle represented by F G E ; whence you have the Species of the Face A F E G. And in the same manner you may find the Species of the other Faces.

DEMONSTRATION.

The opposite Sides of the Figure being to represent Parallels, the Points where they meet must be their Vanishing Points, and consequently the Lines H I, I K, K H will be their Vanishing Lines. But because of all the Angles being Right-angles, H and I will be the Vanishing Points of Lines perpendicular to the Planes whose Vanishing Lines are I K, and H K, and consequently L and M will be the Centers of those Vanishing Lines, and S the Center of the Picture, as is sufficiently evident by *Prop.* 8. But because of the Circle, the Angle I O K is a Right-angle. Wherefore the Distance L O is rightly found to make E G A represent a Right-angle ; whence the Angles N O K, and N O I are found, which are represented by F G A, and F G E, by *Prop.* 12.

G

When

When the Sides A B, G C, E D are parallel to each other, (*n* : 2.) the Point H will be infinitely Diſtant, and the Vaniſhing Line I K will paſs thro' the Center of the Picture, the Points L and S in this caſe co-inciding. But becauſe the Point L in this Caſe is left undetermined, the Species of the Figure is alſo undetermined. So that the Point O may be taken any where in the Semicircle I O K. Whence if the Semicircle I O K be placed perpendicular to the Picture, a Spectator's Eye placed any where in that Circumference, will take the Figure A D for a Right-angled Parallelepipedon ; tho' the Species of it will be different, according to the different Places of the Spectator's Eye. But this is, provided the Lines A B, G C, &c. be perpendicular to the Vaniſhing Line K I ; otherwiſe the Figure can't repreſent a Right-angled Parallelepipedon, but one inclined on a Right-angled Baſe.

I leave it to the Artiſts to conſider whether this Obſervation may not be of Uſe in Painting the Scenes of a Theater. For hence it appears, that if I and K were the two Points, that in the common Books of Perſpective are call'd the Points of Diſtance, and the Buildings in the Scene were ſo drawn, that their Sides ſhould run to thoſe Points, (as in the preſent Figure,) they would always appear Right-angled to the Spectators in in the front Seats of the Boxes, their Eyes being all in Circumference of the Semicircle I O K ; for tho' the Proportions of their Sides were ſomething alter'd, that would be no great Inconvenience.

F I N I S.

AN Attempt towards Recovering an Account of the Numbers and Sufferings of the Clergy of the Church of *England,* Heads of Colleges, Fellows, Scholars, &c. who were Sequester'd, Harrass'd, &c. in the late Times of the Grand Rebellion : Occasioned by the Ninth Chapter (now the Second Volume) of Dr. *Calamy*'s Abridgment of the Life of Mr. *Baxter.* Together with an Examination of that Chapter. By *John Walker,* D. D. Rector of St. *Mary the More* in *Exeter,* and sometime Fellow of *Exeter* College in *Oxford. Folio.* Price 23 *s.*

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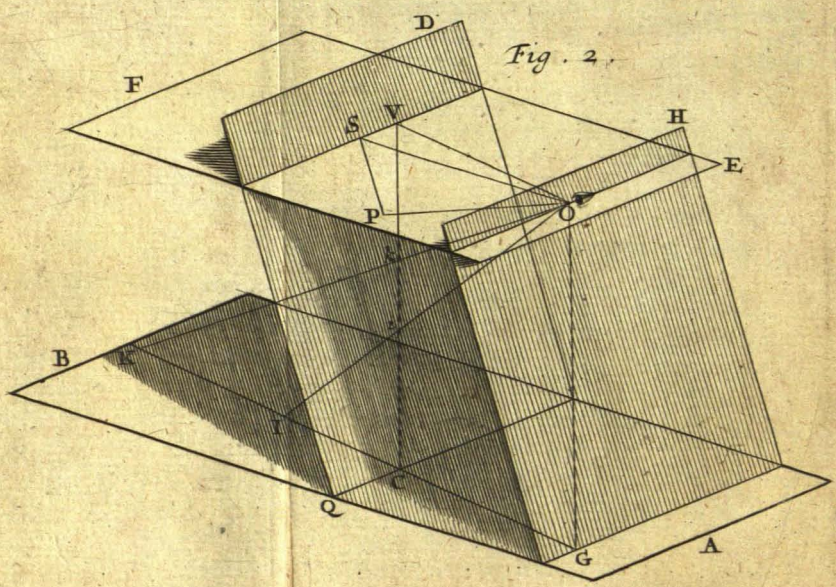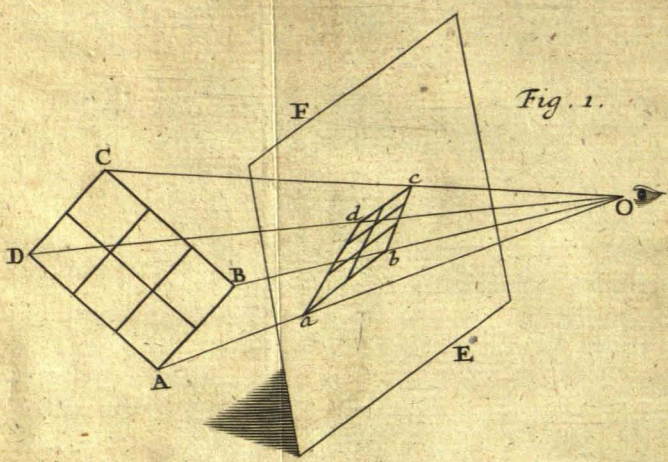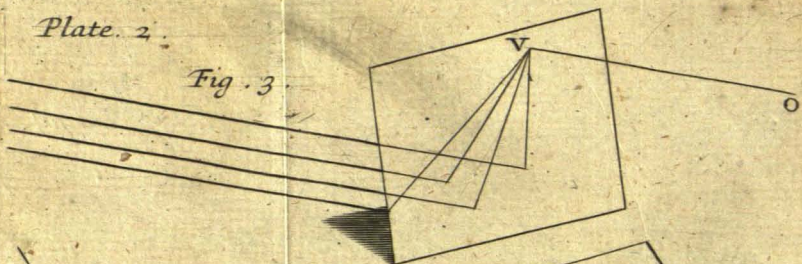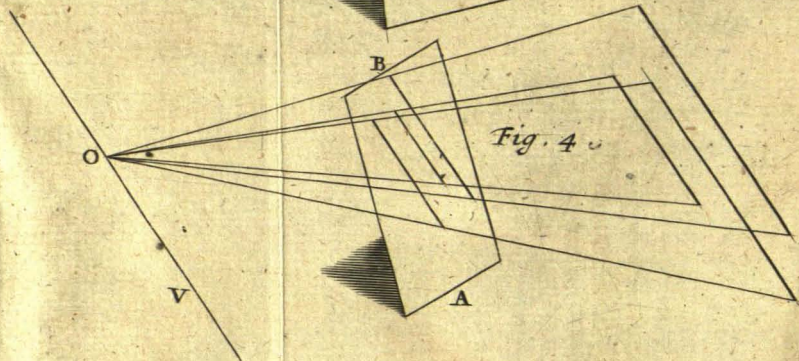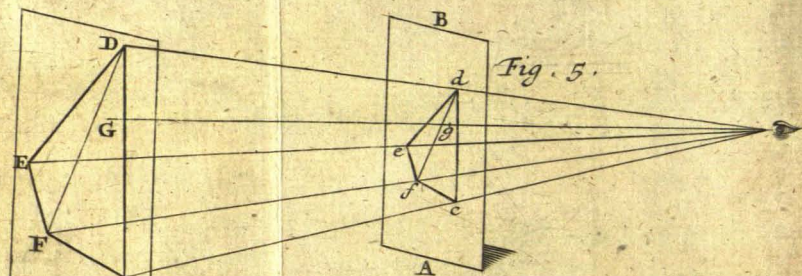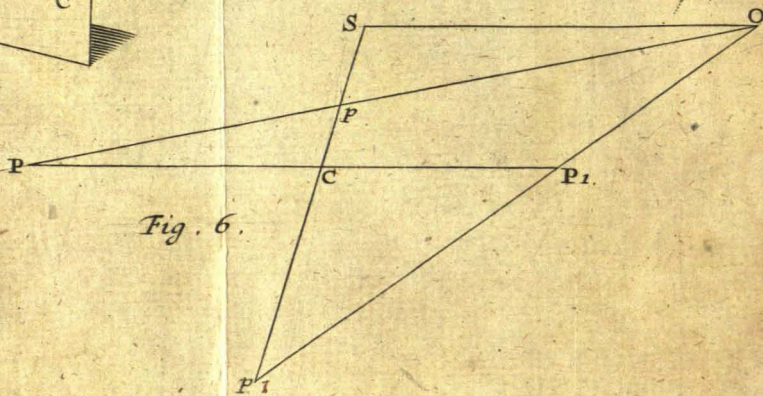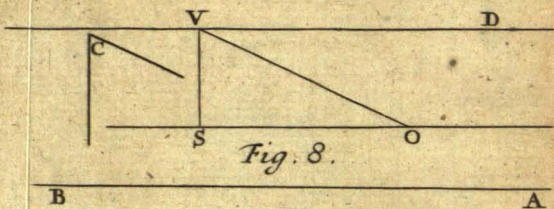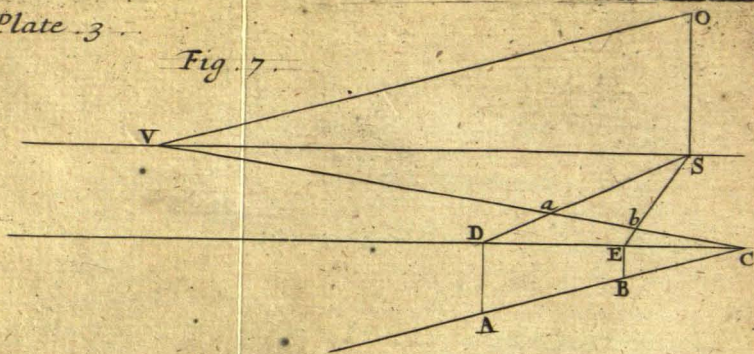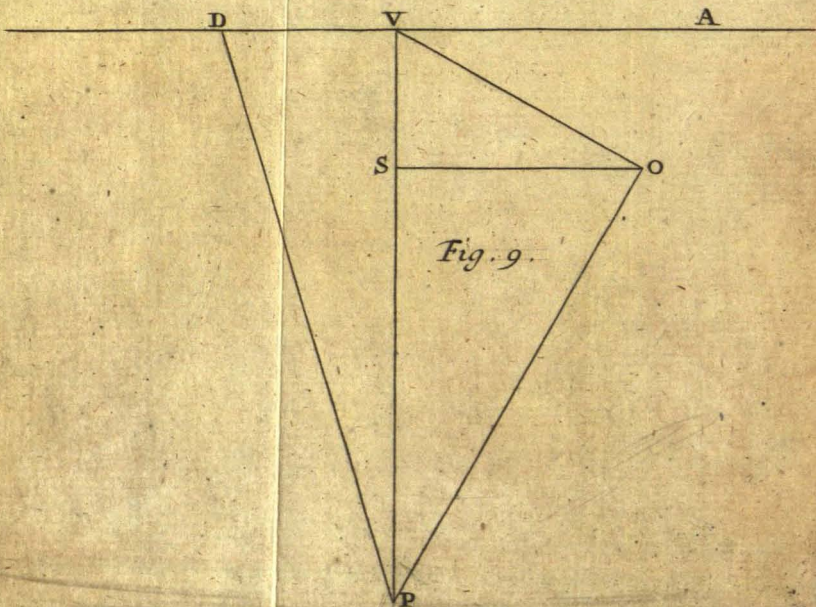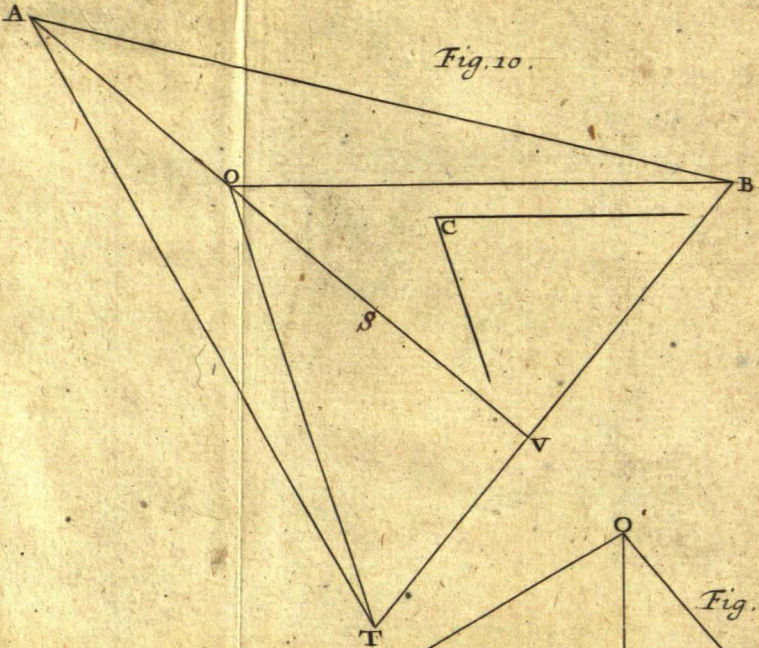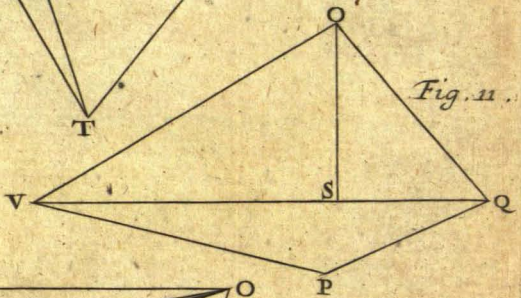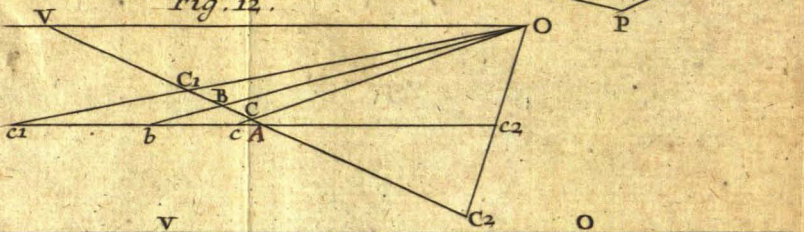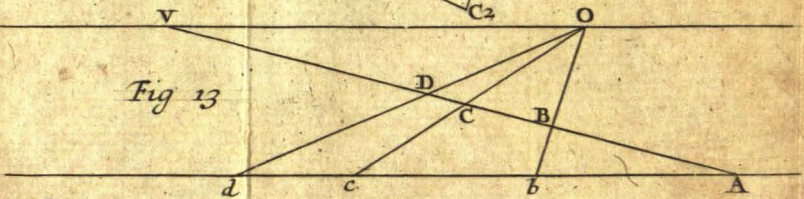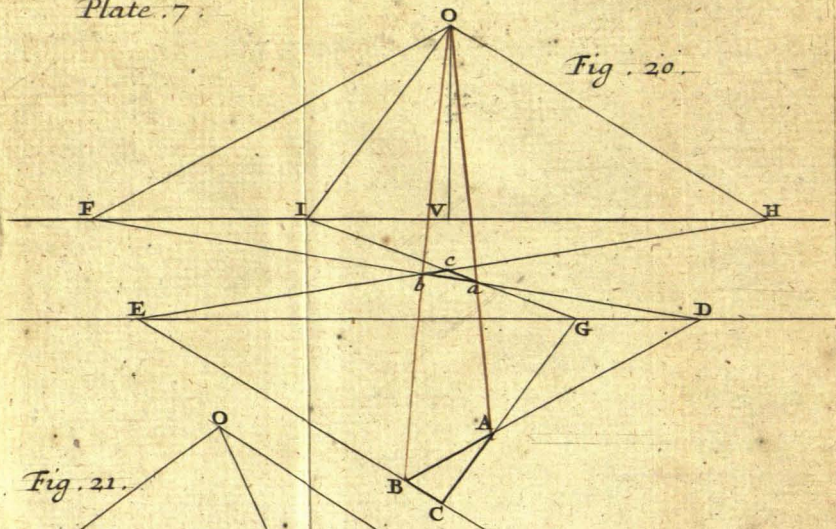
Plate .1.

Fig. 1.

Fig . 2 .

Plate 2.

Fig. 3

O

V

Fig. 4

O

B

V

A

Fig. 5

B

D

d

G

g

E

e

F

f

c

C

A

S

O

P

P

P

C

P₁

Fig. 6.

P₁

Plate 3.

Fig. 7.

Fig. 8.

Fig. 9.

Plate. 4.

Fig. 10.

Fig. 11.

Fig. 12.

Fig. 13

Plate .5 .

Fig. 14 .

Fig. 15 .

Fig. 16 .

Plate. 6.

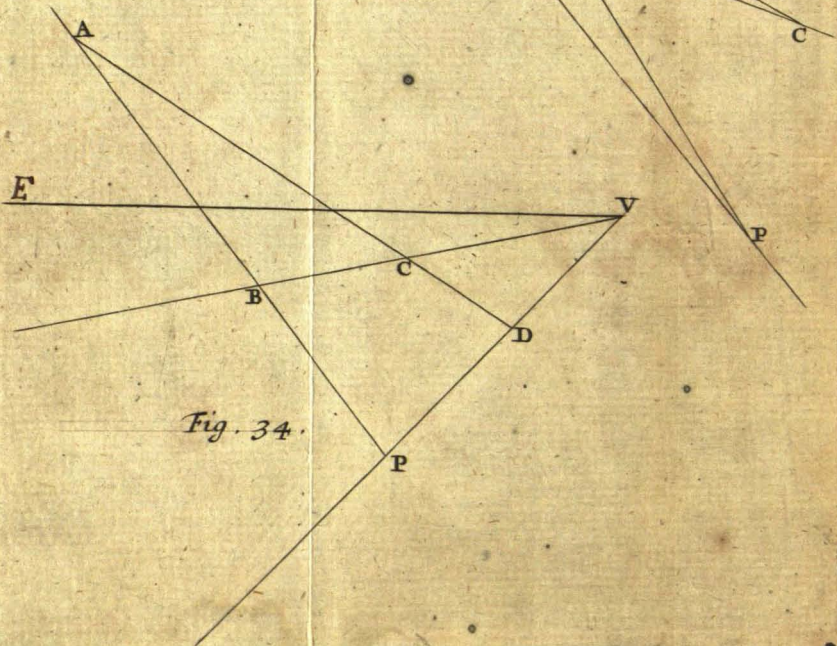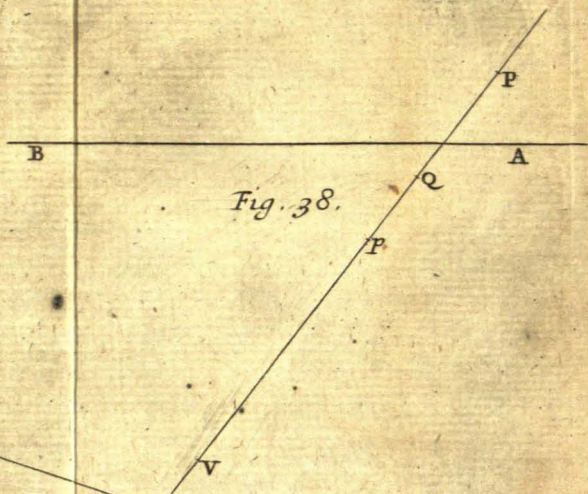Fig. 17.

Fig. 18.

Fig. 19.

Plate 7.

Fig. 20.

Fig. 21.

Fig. 22.

Plate. 8.

Fig. 23

Fig. 24

Plate. 9.

Fig. 25.

Fig. 26.

Fig. 27.

Plate.10.

Fig 28.

Fig. 29.

Plate. 11.

Q P

D S

Fig. 30.

R

V

t T

C O

c

a A

t T

Q q B P B

a

Fig. 31.

A

Plate. 12.

n.1.

B A

P P

D

C

P

A B a P b

D P

C d P

c n.2.

Fig. 32.
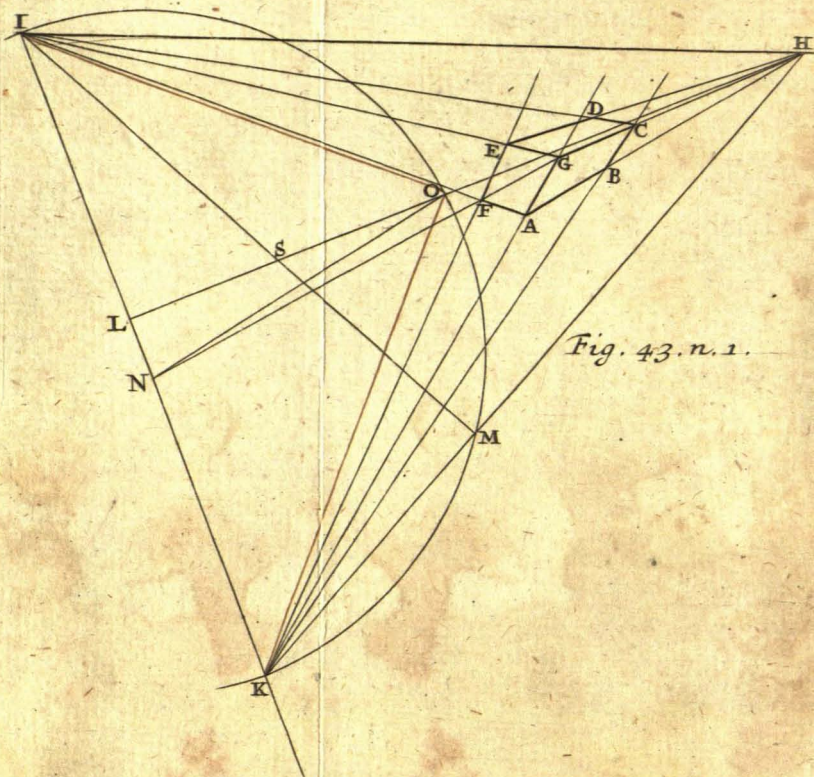
Q

b B a A

Q

D d C c

n.3 P

P

S

T V E

A

c R s B C

D

Fig. 33. n.i.

P

Plate. 13.
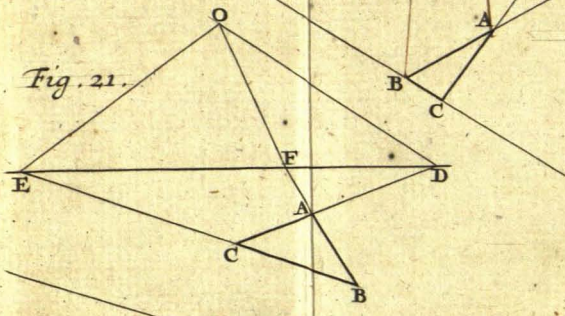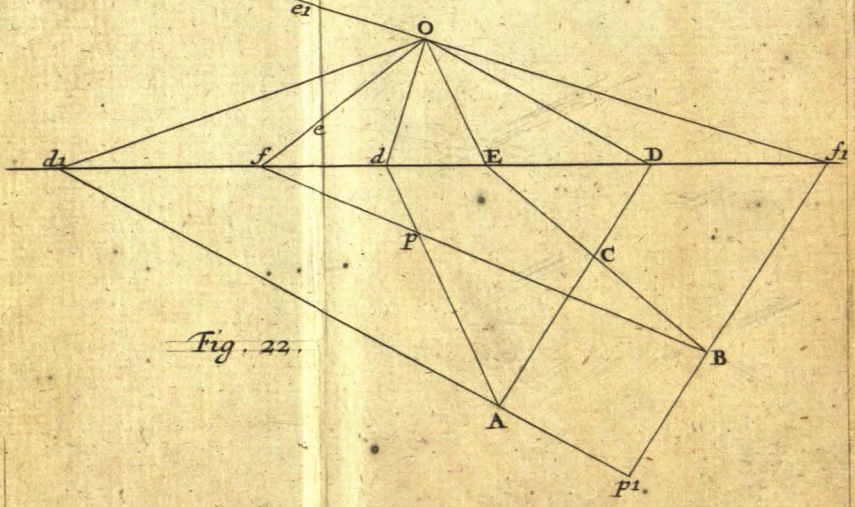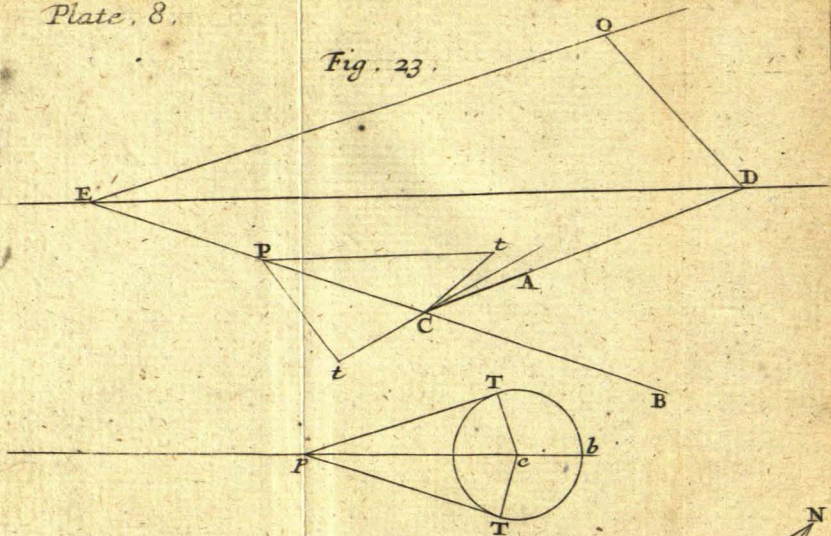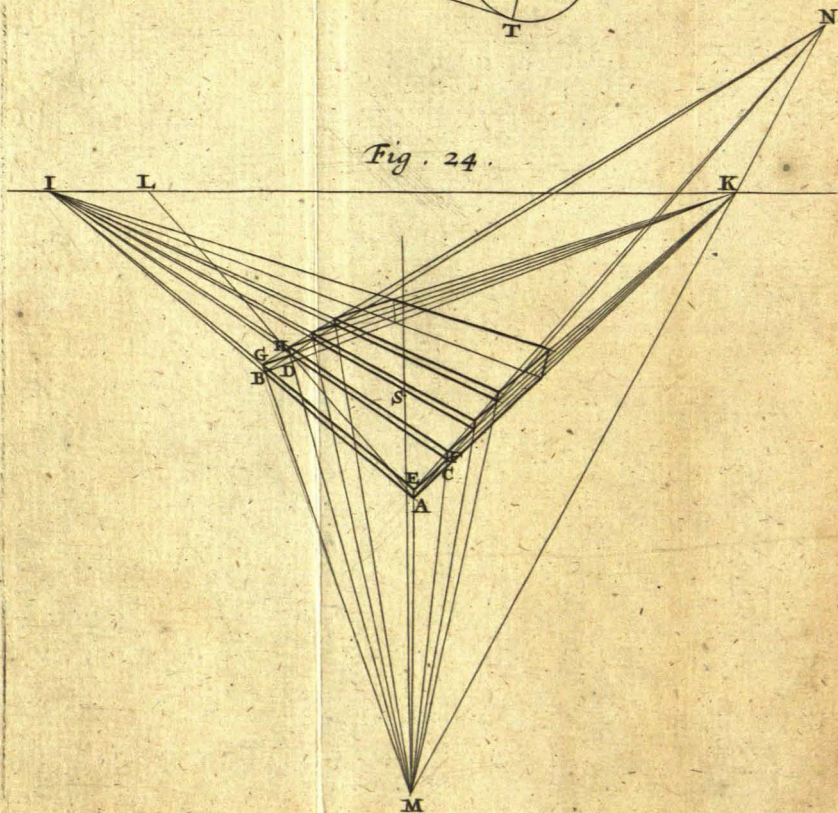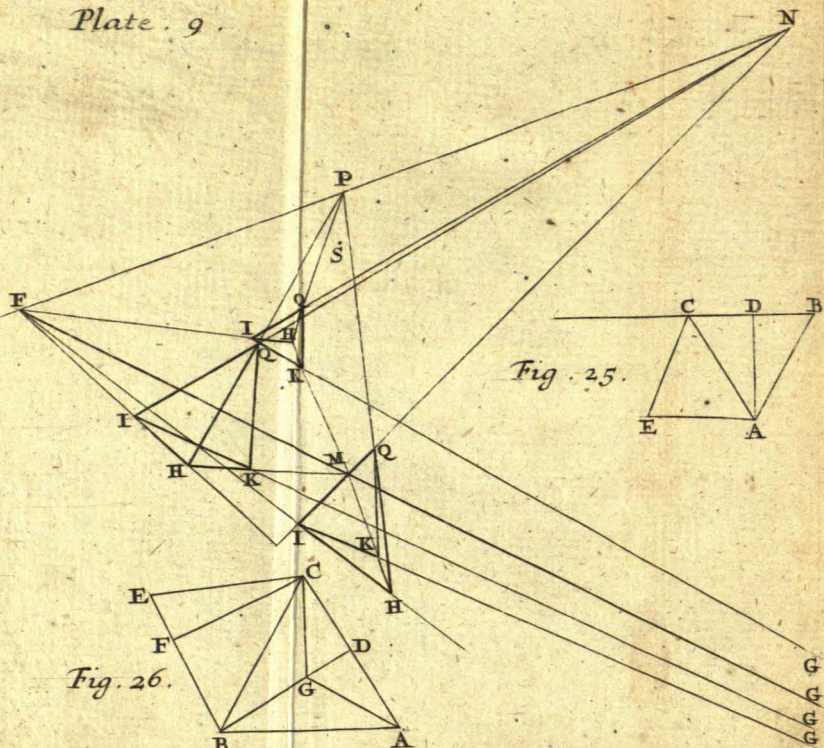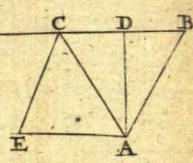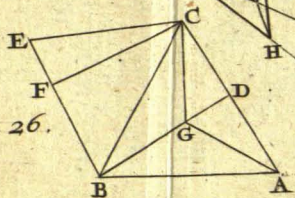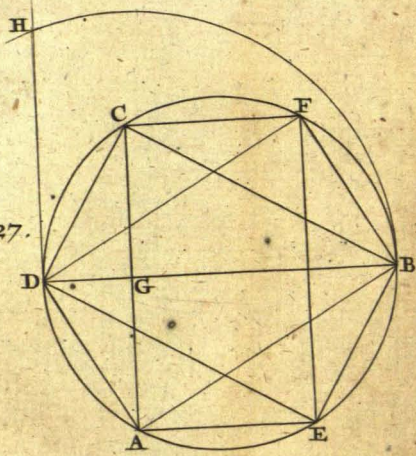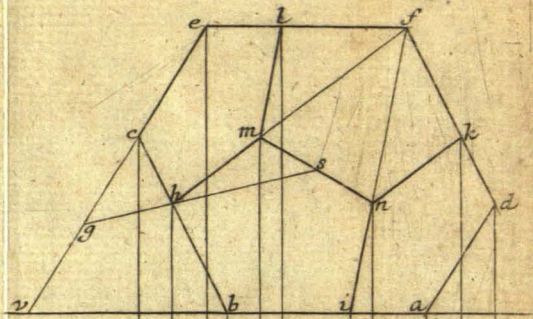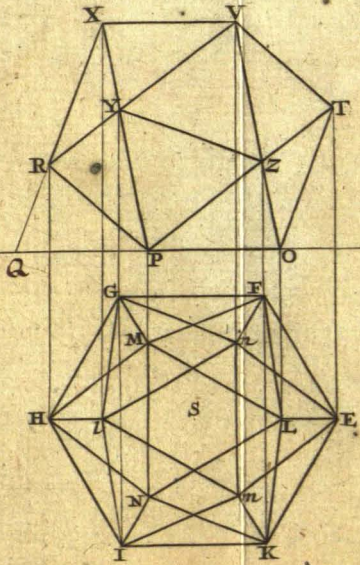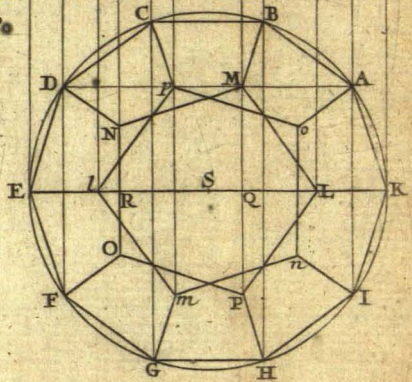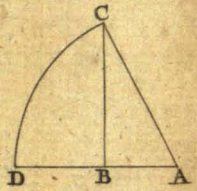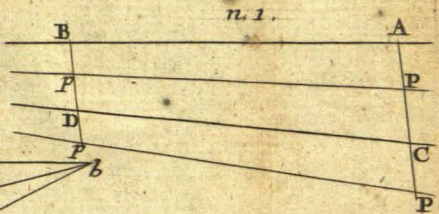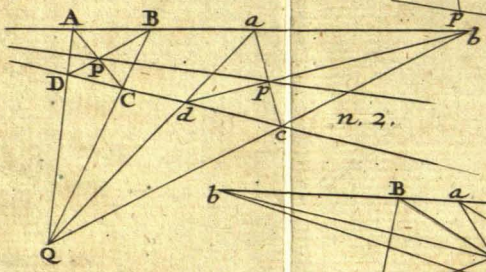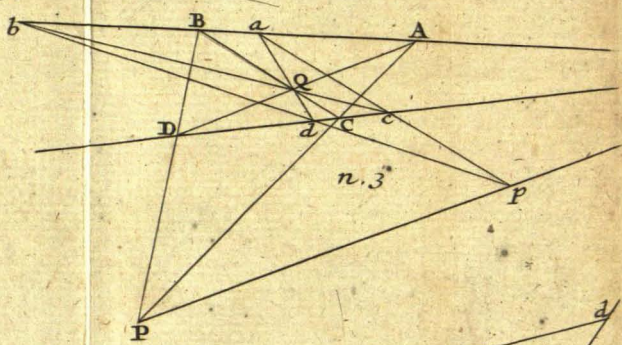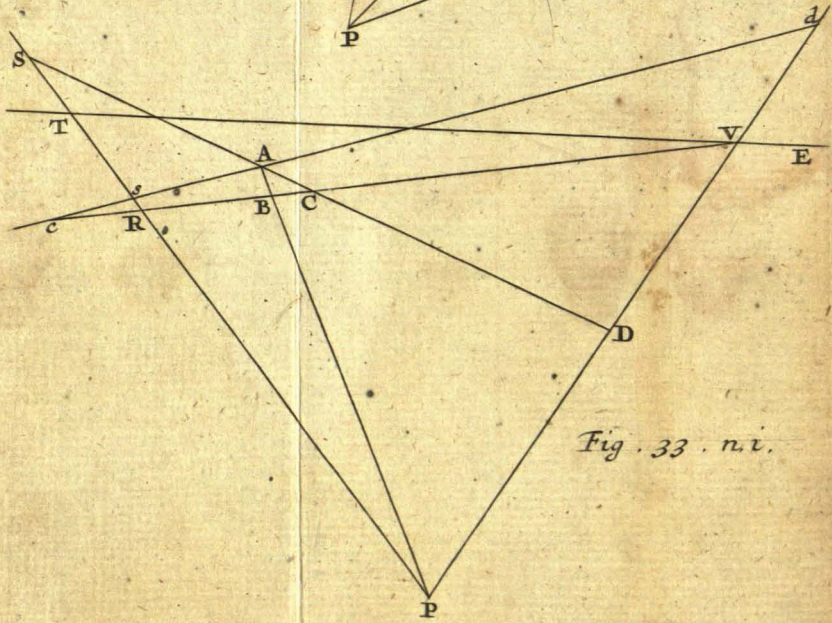
Fig. 33. n. 2.

Fig. 33. n. 4.

Fig. 33. n. 3.

Fig. 34.

Plate. 14.

Fig. 35.

Fig. 36.

Fig. 37.

Plate. 15.

Fig. 38.

Fig. 39.

Plate .16.

Fig. 40.

Fig. 41.

F

C

B

G

E

D

A

Fig. 42.

I

H

D

C

E

G

B

O

F

A

S

L

N

M

Fig. 43. n. 1.

K

Plate. 18.

Fig. 43. n. 2.