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PLATONISM, INTUITION, AND THE NATURE OF MATHEMATICS

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ABSTRACT

Platonist attitude of mathematicians to objects of their investigations is determined by the very nature of the mathematical method. The evolution of greek mathematics led to mathematical objects in the modern meaning of the word: the ideas of numbers, points, straight lines etc. s t a b i l i z e d and thus - were distracted from their real source - properties and relations of things in the real world. Mathematicians do not investigate the nature directly, they investigate some fixed notion of it, and during the investigation this notion is teated (subjectively) as the "last reality" without any "more fundamental" reality behind it. This sort of platonism is an essential aspect of mathematical method. Mathematicians are learned ability " t o l i v e " in the "world" of mathematical concepts. Here we have the main source of the creative power of mathematics, and of its surprising efficiency in natural sciences and technique. In this way, "living" (sometimes - for many years) in the "world"of their concepts and models, mathematicians are learned to draw a maximum of conclusions from a minimum of premises.

A fixed system of basic principles is the distinguishing property of every mathematical theory. A mathematical model of some natural process or technical device is essentially a f i x e d m o d e l which can be investigated independently of its "original". We can change such a model (obtaining a new model) not only for the sake of correspondence to "original", but also for a mere experiment. In this way we obtain easily various "models at themselves". The fixed character of mathematical models makes such deviations possible and even inevitable.

The fixed character of mathematical models is simultaneously the force and the weakness of mathematics: no concrete fixed model (theory) can solve all problems arising in science (or even in mathematics itself). An excellent confirmation of this dialectical thesis was given in the famous incompleteness theorem of K. Goedel.

Only few people will dispute the fixed character of a fully axiomatized theory. But theories, which are not yet axiomatized - can they be fixed, too? Trying to explain this phenomenon, we are led to the concept of i n t u i t i o n . Intuition is treated usually as something like "creative thinking" (see [1, 3]). But in real mathematical theories we have th emost elementary type of intuition - some unconscious "reasonable principles" ruling (together with the axioms, or without any axioms) our reasoning. We can say, therefore, that a theory (or model) can be fixed not only due to some system of axioms, but also due to a specific intuition.

While investigations are going on, they can achieve the level of complexity, at which the degree of definiteness of intuitive models is already insufficient - because of inevitably uncontrollable nature of unconscious processes. The only reliable exit from such situations is following: we must convert (at least partly) the unconscious ruling "principles" into conscious ones. It appeared, however, that - after an explicit reconstruction - some of the concepts possess unexpected properties, missing in the original intuitive concepts. Thus, for example, a continuous function was constructed, which is everywhere nondifferentiable. The appearance of unexpected properties in reconstructed concepts means, that here we have indeed a reconstruction (not a direct "copying" of intuitive concepts).

What criteria can be set for the adequacy of reconstructions? And how, at all, the adequacy of a reconstruction can be founded, if the original concept remains hidden in intuition and every attempt to get it out is a reconstruction itself with the same problem of adequacy? The only possible real answer is: to take into account only those aspects of intuitive concepts, which can be recognized in practice of mathematical reasoning.

Goedel's incompleteness theorem has provoked very much talking about insufficiency of the axiomatic method for a true reconstruction of the "alive, informal" mathematical thinking (see [2]). In fact, it is nonsense to speak about the limited applicability of axiomatics: the limits of axiomatics coincide with the limits of mathematics itself.

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