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K. M. PODNIEKS, *Methodological consequences of Gödel's incompleteness theorem.*
Computing Center, Latvian State University, 226250 Riga, USSR.

The usual definition of "true" formulas in first-order arithmetic (inductively, by structure of formulas) provokes Platonist illusions (see [1]) that any closed formula must be either "true" or "false". But it is impossible to verify a formula like $(Ax)C(x)$ empirically. This can be done only *theoretically*, i.e., using some fixed system of axioms.

Similar illusions are connected with the categoricity theorem for second-order arithmetic. This theorem can be proved within ZF; however, it does not yield that any closed formula of arithmetic can be proved or disproved in ZF.

The latter illusion is connected with some features of Gödel's proof (see [2]) of his incompleteness theorem: Gödel's formula $G(T)$, constructed for some theory T, "is true, but not provable". But the truth of $G(T)$ can be established only by postulating the consistency of T, and since $\mathrm{Con}(T) \rightarrow G(T)$ can be proved (for any T) in first-order arithmetic, we should not speak about the "informal truth" of $G(T)$.

The real meaning of the incompleteness theorem is the following: any serious theory based on a fixed system of principles cannot be made *perfect*—it inevitably contains either contradictions or undecidable problems. The mere postulating of the excluded middle $(F \wedge \neg F)$ in some theory T does not yield the decidability of all closed formulas F in T.

Any theory (in mathematics, science, or other branches of intellectual life) is essentially a construction involving inevitably many elements of fantasy (even the arithmetic of natural numbers contains such elements; see [3]). The incompleteness theorem says that no fantastic construction can be designed "logically" enough to ensure the decidability of all definite statements of it.

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