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Fuzzy Logic Based Mathematical Structures and Their Applications

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University of Latvia

Contents

An Alternative Approach to Fuzzification of a Rough Set

ANTOŅUKS Aleksejs

University of Latvia, Department of Mathematics
Jelgavas iela 3, Riga, LV-1004, Latvia
E-mail: aleksey.antonyuk1@gmail.com

We show an alternative approach to fuzzification of a rough set. The method of approach is based on α-equivalence classes. Rough set theory was developed as a mathematical tool that allows to deal with big volumes of data and later found apllication in domains such as medicine, marketing, image analysis, etc. (for detailed information see [3], [2]). Fuzzy rough set appeared first in the paper [1]. Our aim is to introduce an alternative method of fuzzification. To show results some examples are considered.

References:

[1] Dubois D., Prade H.: *Rough Fuzzy Sets and Fuzzy Rough Sets.* International Journal of General Systems, **17** (1990) 191–209.

[2] Komorowski J., Pawlak Z., Polkowski L., Skowron A.: *Rough Sets: A Tutorial.* (1998), URL http://bioinf.icm.uu.se/kbib/RoughSets-Tutorial.pdf.

[3] Nasiri J. H., Mashinchi M.: *Rough Set and Data Analysis in Decision Tables.* Journal of Uncertain Systems, **3** (2009) 232–240.

Fuzzy Approximation Methods in Combinatorics on Words [1]

BĒTS Raivis

Institute of Mathematics and Computer Science, University of Latvia
Raina bulvaris 29, Riga, LV-1459, Latvia
E-mail: raivis.bets@lu.lv

Based on the definition of a probabilistic metric, I. Kramosil and J. Michálek [1] introduced the notion of a fuzzy metric called KM-fuzzy metric. On the basis of KM-fuzzy metric, A. George and P. Veeramani [2] introduced an alternative concept of a fuzzy metric, known now as a fuzzy metric in the sense of George and Veeramani or GV-fuzzy metric.

We construct three kinds of parameterized and fuzzy approximating pseudometrics. In the first approach we readjust the definition of a GV-fuzzy pseudometric and define a strong fuzzy approximating pseudometric [3]. We give some numerical results based on the change of the prefixes. In the second approach we construct a parameterized metric using a t-conorm [4]. In the third approach we choose a specific function, which satisfies the axioms of parameterized function and gives good results when comparing two infinite and periodic words.

References:

[1] Kramosil I., Michálek J.: *Fuzzy Metrics and Statistical Metric Spaces.* Kybernetika, **11**, (1975), 336–344.

[2] George A., Veeramani P.: *On Some Results in Fuzzy Metric Spaces.* Fuzzy Sets Syst., **64** (1994), 395–399.

[3] Bēts R., Šostak A.: *Some Remarks on Strong Fuzzy Metrics and Strong Fuzzy Approximating Metrics with Applications in Word Combinatorics.* Mathematics, **10(5)**, (2022), 738.

[4] Bēts R., Šostak A., Miķelsons E.M.: *Parameterized Metrics and Their Applications in Word Combinatorics.* In: Information Processing and Management of Uncertainty in Knowledge-Based Systems. IPMU 2022. Communications in Computer and Information Science, Springer, **1601** (2022).

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Fuzzy Metrics: Back to Roots [2]

GRIGORENKO Olga

Institute of Mathematics and Computer Science, University of Latvia
Raina bulv. 29, Riga, LV-1459, Latvia
E-mail: ol.grigorenko@gmail.com

Nowadays in literature the notion of a fuzzy metric is basically used under the axioms introduced in [1] and [2]. These axioms are actually a reformulation of the axioms originally defined in [3]. In [3] the idea for a definition of a fuzzy metric comes from the assertion that the considered value of a crisp metric $d(x, y)$ (which should be fuzzified or approximated) is smaller than a given real number λ. In other words, the statement $d(x, y) < \lambda$ is fuzzified. In the work we explain how the idea of a fuzzy metric has been arisen from a crisp order relation, first showing that each metric d could be determined by an order R_d, and then fuzzyfying the axioms of R_d. We investigate which conditions should a fuzzy relation fulfill to be a fuzzy metric. We also mention the idea of the construction of a fuzzy metric by fuzzifying the statement $d(x, y) = \lambda$.

References:

[1] George A., Veeramani P.: *On Some Results in Fuzzy Metric Spaces.* Fuzzy Sets Syst., **64** (1994) 395–399.

[2] Grabiec M.: *Fixed Points in Fuzzy Metric Spaces.* Fuzzy Sets Syst., **27** (1988) 385–389.

[3] Kramosil I., Michálek J.: *Fuzzy Metrics and Statistical Metric Spaces.* Kybernetika, **11** (1975) 336–344.

[2] **Acknowledgement** This work is supported by European Regional Development Fund within the PostDoc Latvia project Nr.1.1.1.2/16/I/001, under agreement Nr.1.1.1.2/VIAA/4/20/707 "Fuzzy relations and fuzzy metrics for customer behavior modeling and analyis".

Fuzzy Categories

KALUGINS Emīls

University of Latvia, Department of Mathematics
Jelgavas iela 3, Riga, LV-1004, Latvia
E-mail: emils.kalugins@lu.lv

Ever since the inception of fuzzy set theory, many concepts in classical mathematics have been adapted to a fuzzy analogue. Numerous crisp categories have been researched in which fuzzy sets are involved in one way or another. However, a category of fuzzy structures is not a fuzzy structure itself. Here we will consider the concept of a fuzzy category which was first introduced by Alexander Šostak and further developed by him in [1].

We give an overview of the basic concepts related to fuzzy categories such as a subcategory, a quotient category, a monomorphism, an epimorpshim and a functor. After this brief introduction numerous examples are given, illustrating how fuzzy categories can arise from certain classic categories such as the category of sets, groups, topological spaces, proximity spaces in a natural way [1], [2]. To conclude, we discuss the future prospects of fuzzy categories. Even though many of the basic concepts of category theory have been developed, the notion of a fuzzy natural transformation is yet to be defined. We also discuss the alternative approach to fuzzy categories using fuzzy graph theory introduced in [3].

References:

[1] Šostak A.: *Fuzzy Categories Related to Algebra and Topology.* Fuzzy Sets: Theory and Appl., **12** (1999) 159–185.

[2] Šostak A.: *On Some Fuzzy Categories Related to the Category L-TOP of L-topological Spaces.* Topological and Algebraic Structures in Fuzzy Sets, **20** (2003) 211–240.

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Fuzzy Equivalences and Inequality Relations

KLIMANOVA Jelizaveta

University of Latvia, Department of Mathematics
Jelgavas iela 3, Riga, LV-1004, Latvia
E-mail: lizavetakli@gmail.com

In this work we present a brief overview of the most popular triangular norms and conorms. We construct t-norms using additive generators and introduce fuzzy equivalences that are constructed based on t-norms. Furthermore, we define fuzzy inequality relations using fuzzy equivalences and involution.

On Application of Fuzzy Concept Lattices in Comparison of Different Datasets

KRASTIŅŠ Māris

University of Latvia, Department of Mathematics
Jelgavas iela 3, Riga, LV-1004, Latvia
E-mail: mk18032@edu.lu.lv

Formal concept analysis originally introduced by Bernhard Ganter and Rudolf Wille, see, e.g. [2], provides a theoretical basis for mathematical research of objects and their properties. The basic notion of the formal concept analysis is a formal context, that is a triple (X, Y, R), where X is a set, whose elements are interpreted as objects, Y is a set, whose elements are interpreted as properties, and $R \subseteq X \times Y$ is a relation, where the entry xRy is interpreted as "an element x has property y". The goal of concept analysis is to reveal pairs (A, B), where sets $A \subseteq X$ of objects and sets $B \subseteq Y$ of properties are mutually connected by the relation R.
In the fuzzy case, a context is a tuple (X, Y, L, R), where X and Y are non-empty sets, L is a lattice, and $R : X \times Y \to L$ is an L-fuzzy relation. Fuzzy concepts in this fuzzy context are pairs (A, B), where A and B are L-fuzzy subsets of the sets X and Y, respectively. The most important work in the first decade of the 21st century in the field of fuzzy concept analysis was carried out by R. Bělohlávek; see, e.g. [1].
As a part of our research we have introduced fuzzy preconcepts, see [3], in order to avoid certain limitations related to fuzzy concept lattices. We analyze so called object and property oriented fuzzy preconcepts. For the purposes of data comparison we propose replacement of set Y, representing properties, with set Y' representing other objects. Such task is natural in cases when there is a need to find the objects, e.g., concrete persons in the external database (this is a typical example of screening of persons against international sanctions lists). First we consider matching of names by applying traditional Levenshtein distance algorithm. Under such scenario fuzzy relation R reveals the level of match for concrete person against similar persons from external database. Furthermore we apply additional criteria to the objects X and Y' in a such way that they characterize completeness of person's data (apart from the person's name we consider its date of birth, personal ID number). As the result, we have obtained the model consisting of both fuzzy objects X and Y' as well as fuzzy relations R. This model can be further used to study the problems of data assessment and comparison, e.g., assessment of quality of sanctions lists and screening of selected databases against the sanctions lists.

References:

[1] Bělohlávek R.: *Concept Lattices and Order in Fuzzy Logic.* Annals of Pure and Applied Logic, **128** (2004) 277–298.

[2] Ganter B., Wille R.: *Formal Concept Analysis: Mathematical Foundations.* Springer Verlag, Berlin, 1999.

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Fuzzification of Some Machine Learning Methods [3]

MIHAILOVS Valerijs and GRIGORENKO Olga

Institute of Mathematics and Computer Science, University of Latvia
Raina bulv. 29, Riga, LV-1459, Latvia
E-mails: tykon@inbox.lv, ol.grigorenko@gmail.com

The aim of the work is to involve fuzzy equivalence relations and aggregation of corresponding equivalence relations in machine learning methods. Namely, in clustering process we introduce fuzzy equivalence relations for different attributes of objects and then we aggregate these fuzzy equivalence relations to determine the similarities of objects [1]. It is possible to involve different weights for attributes, thus defining the importance attributes in decision making process. We will see, how equivalence relations simplify a cost function calculation. Also, we will consider possibility to use fuzzy equivalence relations in other machine learning methods. In addition, we will present illustrative examples.

References:

[1] Grigorenko O., Mihailovs V.: *Aggregated Fuzzy Equivalence Relations in Clustering Process.* CCIS, **1601** (2022) 448–459.

[3] **Acknowledgement** This work is supported by European Regional Development Fund within the PostDoc Latvia project Nr.1.1.1.2/16/I/001, under agreement Nr.1.1.1.2/VIAA/4/20/707 "Fuzzy relations and fuzzy metrics for customer behavior modeling and analyis".

On Two Categories of Fuzzy Morphological Spaces

VARFOLOMEJEVA Ksenija

University of Latvia, Department of Mathematics
Jelgavas iela 3, Riga, LV-1004, Latvia
E-mail: ksyvarf@gmail.com

In our work we investigate the possibilities of representing the operators of classical mathematical morphology – dilation and erosion as special categories of spaces. We construct the basis of our approach in classical rough set theory [1] and then provide the idea of extending the theory to fuzzy spaces [2]. First, we create morphological structures based on the results of exposure to both morphological operators on a set, and second, we extend the application to the power set. We explore the second morhological category using the diagram approach, introducing derived morphological operators – closing and opening.

References:

[1] Nachtegael N., Kerre E.E.: *Classical and Fuzzy Approaches towards Mathematical Morphology.* Studies in Fuzziness and Soft Computing, (2000) 27–43.

[2] Šostaks A.: *L-kopas un L-vērtīgas struktūras.* Riga, 2003.

Clusterization of Pareto Frontier in MOLP Problems

ZEMLĪTIS Mārtiņš

University of Latvia, Department of Mathematics
Jelgavas iela 3, Riga, LV-1004, Latvia
E-mail: martins.zemlitis@gmail.com

In this work multiple ways of cluster analysis for Pareto front are looked at. The aim of the cluster analysis is to find a way to reduce the size of the Pareto front in a multi-objective linear programming problem with two objective functions. When solving multi objective linear programming problems using fuzzy orderings [1], a pseudo-metric was introduced, to measure distance between two points using objective functions. This metric was aggregated along the Pareto front and then fuzzy c-means algorithm was used on resulted one-dimensional array using Euclidean distance or the clusterization was done on Pareto front using fuzzy c-means where distance in algorithm was calculated using this pseudo-metric. Both results gave similar results, although in examples, where objective functions compensate each other, first approach fails to give a result.

References:

[1] Grigorenko O.: *Involving Fuzzy Orders for Multi-Objective Linear Programming.* Mathematical Modelling and Analysis, **17(3)** (2012) 366–382.